

THERMOMETRY

ZEROTH LAW OF THERMODYNAMICS :

If an object A is in thermal equilibrium with object B and the object B is in thermal equilibrium with object C then A is in thermal equilibrium with C . The Zeroth law gives the basis for the temperature measurement.

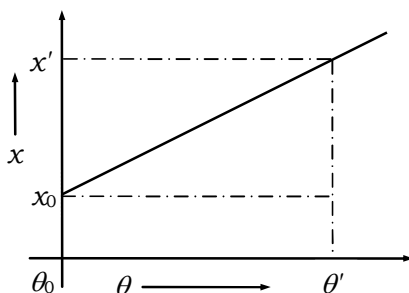
Temperature :

The temperature is a physical quantity characterizing the degree of hotness or coldness of a body. If a system is in a state of thermodynamic equilibrium all bodies constituting the system are at the same temperature. The temperature can only be measured indirectly making use of the fact that certain physical properties of bodies lending themselves to direct measurement. The bodies employed for this purpose are said to be thermometric, their property indicating temperature is called thermometric property and the temperature scale based on them is said to be empirical.

Relation between different scales of temperature

$$\frac{C}{5} = \frac{F - 32}{9} = \frac{K - 273}{5} = \frac{R}{4}$$

If x is a thermometric property which changes linearly with temperature then



$$\Delta x \propto x_0$$

and

$$\Delta x \propto \Delta \theta$$

hence

$$\Delta x \propto x_0 \Delta \theta$$

$$\Rightarrow \Delta x = \alpha x_0 \Delta \theta$$

here α , the constant of proportionality is known as temperature coefficient of thermometric property. Therefore

$$x_\theta - x_0 = \alpha x_0 \Delta \theta$$

$$\Rightarrow x_\theta = x_0 (1 + \alpha \Delta \theta)$$

... (1)

If $\theta_0 = 0$ then $\Delta \theta = \theta - \theta_0 = \theta$ hence

$$x_\theta = x_0 (1 + \alpha \theta)$$

As x varying with θ linearly with (x', θ') and (x_0, θ_0) being two points on $x - \theta$ graph to fit the straight line then

$$\theta = \frac{x_\theta - x_0}{(x' - x_0)} \times (\theta' - \theta_0)$$

If value of thermometric property at two different temperatures θ_1 and θ_2 is x_1 and x_2 then from equation (1)

$$x_1 = x_0(1 + \alpha\theta_1) \text{ and}$$

$$x_2 = x_0(1 + \alpha\theta_2)$$

dividing we get

$$\alpha = \left\{ \frac{x_2 - x_1}{\theta_2 x_1 - \theta_1 x_2} \right\}$$

CALORIMETERY :

Specific heat of liquids and solids :

It is the amount of energy required to increase the temperature of the unit mass of substances by 1°C .

$$s = \frac{dQ}{m d\theta}$$

Total heat capacity :

Is the amount of heat required to increase the temperature of the entire object by 1°C .

$$C = ms.$$

Water equivalent :

The amount of water, when given same amount of heat shows same change in temperature

$$w = m s = C.$$

It is numerically equal to the heat capacity of the object.

Latent heat :

The amount of Energy to change the state of unit mass of a substance.

$$\ell = \frac{dQ}{m}$$

Here we have to apply the law of conservation of energy. The simple rule is energy given is same as energy taken.

When two or more substances are mixed and chemical changes do not occur then the temperature of mixture cannot cross the boundary.

THERMAL EXPANSION :

When a substance is heated, the average kinetic energy of its molecules and/or atoms increases due to which the average distance between the atoms/molecules increases. This leads to the expansion of whole body. The phenomenon is called thermal expansion. We can talk about

- i) Linear expansion (Coefficient of linear expansion is α)
- ii) Superficial expansion (Coefficient of superficial expansion is β)
- iii) Volume expansion (Coefficient of volume expansion is γ)

If dL is the linear expansion of rod of length L due to increase in temperature $d\theta$ then,

$$dL \propto L \text{ and } dL \propto d\theta \text{ therefore}$$

$$dL = \alpha L(d\theta)$$

$$\Rightarrow \int_{L_0}^L \frac{dL}{L} = \int_{\theta_0}^{\theta} \alpha \cdot d\theta$$

$$\Rightarrow \ln\left(\frac{L}{L_0}\right) = \alpha(\Delta\theta)$$

$$\Rightarrow \ln\left\{1 + \left(\frac{L}{L_0} - 1\right)\right\} = \left\{\frac{L}{L_0} - 1\right\} = \alpha(\Delta\theta)$$

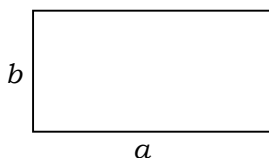
as ΔL is very small therefore neglecting higher order terms.

$$\Delta L = L_0 \alpha \Delta\theta$$

$$\text{therefore } \alpha = \frac{\Delta L}{L_0 \Delta\theta}, \beta = \frac{\Delta A}{A_0 \Delta\theta} \text{ and } \gamma = \frac{\Delta V}{V_0 \Delta\theta}$$

Relation between α , β and γ :

Consider a rectangular sheet of sides a and b . The area of sheet is



$$S = ab$$

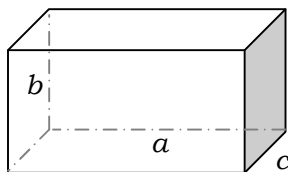
$$\Rightarrow \ln S = \ln a + \ln b$$

Differentiating w.r.t. temperature, we get

$$\frac{1}{S} \frac{dS}{d\theta} = \frac{1}{a} \frac{da}{d\theta} + \frac{1}{b} \frac{db}{d\theta}$$

$$\Rightarrow \beta = \alpha + \alpha = 2\alpha$$

Similarly consider a cuboid of sides a , b and c . The volume of cuboid is



$$V = abc$$

$$\Rightarrow \ln V = \ln a + \ln b + \ln c$$

Differentiating w.r.t. temperature, we get

$$\frac{1}{V} \frac{dV}{d\theta} = \frac{1}{a} \frac{da}{d\theta} + \frac{1}{b} \frac{db}{d\theta} + \frac{1}{c} \frac{dc}{d\theta}$$

$$\Rightarrow \gamma = \alpha + \alpha + \alpha = 3\alpha$$

Hence α , β and γ are related to each other with a simple relation. $\alpha : \beta : \gamma = 1 : 2 : 3$

Variation of density with temperature :

Density of a substance

$$\rho_0 = \frac{m}{V_0}$$

At a temperature θ ,

$$\rho_\theta = \frac{m}{V_\theta} = \frac{m}{V_0(1 + \Delta\theta)}$$

Hence the density of the material changes with temperature as

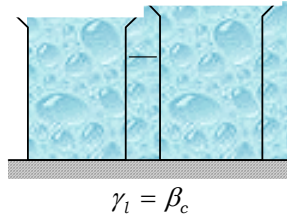
$$\rho_\theta = \frac{\rho_0}{(1 + \gamma\Delta\theta)} \approx \rho_0 (1 - \gamma\Delta\theta)$$

In case of gases we define, coefficient of volume expansion (γ_v) and coefficient of pressure expansion (γ_p).

$$\gamma_v = \left(\frac{\Delta V}{V \Delta\theta} \right)_{P = \text{constant}} \quad \& \quad \gamma_p = \left(\frac{\Delta P}{P \Delta\theta} \right)_{V = \text{constant}}$$

Height of liquid in a container :

Let a liquid with coefficient of volume expansion γ_l is kept in a container of material with coefficient of superficial expansion β_c . If the height of liquid inside container is h_0 and base area A_0 at the temperature $\theta = 0$.



Initially the volume of liquid is

$$V_0 = A_0 h_0$$

When the system is heated to a temperature θ then

$$V_\theta = A_\theta h_\theta$$

$$\Rightarrow h_\theta = \frac{V_0 (1 + \gamma_l \theta)}{A_0 (1 + \beta_c \theta)}$$

$$\Rightarrow h_\theta = h_0 \frac{(1 + \gamma_l \theta)}{(1 + \beta_c \theta)}$$

$$\Rightarrow h_\theta \approx h_0 \{1 + (\gamma_l - \beta_c) \theta\}$$

for vertical height of liquid in the container to remain unchanged

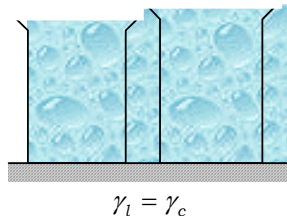
$$h_\theta = h_0$$

$$\Rightarrow \gamma_l = \beta_c$$

But if the container is filled to the top then liquid will not overflow if the thermal expansion in volume of the liquid is same as the expansion of the containing volume of the container. Or else if the container is partially filled and the liquid level is marked inside the container then for

$$\gamma_l = \gamma_c$$

the liquid will maintain the level of mark inside the container although the height of mark inside the container will raise due to thermal expansion of the container.



Thermal Stress :

When a rod is held between two rigid supports and the temperature is allowed to rise, the rigid supports prevent the rod from expanding, this is the cause of a stress produced in the rod which opposes the effect of the temperature change.

$\Delta L = \alpha L \Delta\theta$, ΔL is the thermal expansion

$\Delta L = \frac{FL}{AY}$, ΔL is the mechanical expansion

$$\alpha L \Delta\theta + \frac{FL}{AY} = 0$$

$$\Rightarrow \frac{F}{A} = -\alpha Y \Delta\theta$$

Representative example 1 :

A pendulum clock made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate that averages 30°C, what correction is necessary at the end of 30 days to the time given by clock? $\alpha_{\text{invar}} = 0.7 \times 10^{-6}/^{\circ}\text{C}$

Solution :

The time period of a simple pendulum is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

taking log and differentiating

$$\ln T = \ln(2\pi) + \frac{1}{2} \ln(L) - \frac{1}{2} \ln(g)$$

$$\frac{dT}{T} = \frac{1}{2} \frac{dL}{L}$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{\Delta L}{L} = \frac{\Delta t}{t} \text{ since } \Delta L \text{ is very small compared to } L$$

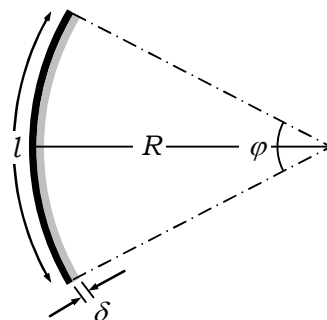
$$\Rightarrow \Delta t = t \times \frac{\Delta L}{2L} = \frac{\alpha \Delta\theta}{2} t$$

$$\Rightarrow \Delta t = \frac{7 \times 10^{-6} \times 10}{2} (30 \times 24 \times 60 \times 60) = 9.072 \text{ sec.}$$

Bending of bimetallic Strip :

Two metal strips of equal length l and thickness δ , with different coefficient of linear expansion α_1 and α_2 are welded together as shown. The system is heated through $\Delta\theta$. From figure,

$$L = R\phi$$



take log and differentiate

$$\ln L = \ln R + \ln \phi$$

$$\Rightarrow \frac{dL}{L} = \frac{dR}{R}$$

as ΔL is very small

$$\frac{\Delta L}{L} = \frac{\Delta R}{R}$$

$$\Rightarrow \frac{L(\alpha_1 - \alpha_2)\Delta\theta}{L} = \frac{\delta}{R}$$

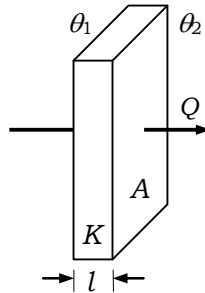
$$\Rightarrow R = \frac{\delta}{(\alpha_1 - \alpha_2)\Delta\theta}$$

HEAT TRANSFER :

There are three modes of heat transfer.

Conduction :

Is that form of heat transfer (mostly through solids or liquids) in which the heat energy is transferred by virtue of temperature difference between two surfaces, without actually transferring the material. In steady thermal conduction state, the rate of heat flow



$$\frac{dQ}{dt} \propto A \text{ (area of cross section) and}$$

$$\frac{dQ}{dt} \propto \frac{\Delta\theta}{l} \text{ (temperature gradient)}$$

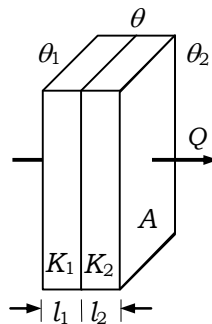
$$\Rightarrow \frac{dQ}{dt} = \frac{KA(\Delta\theta)}{l} = h \text{ (also called thermal current)}$$

and

$$\frac{\Delta\theta}{h} = \frac{l}{KA} = R_r \text{ called thermal resistance}$$

K is the coefficient of thermal conduction.

Rate of heat flow through a composite slab.



$$\frac{dQ}{dt} = \frac{A(\Delta\theta)}{\sum \frac{l}{K}}$$

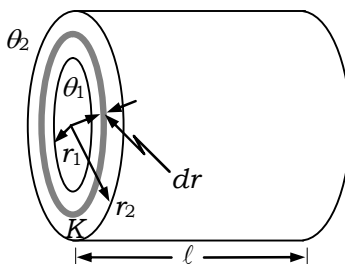
For two slabs, let the interface temperature be θ then

$$\frac{dQ}{dt} = \frac{K_1 A (\theta_1 - \theta)}{l_1} = \frac{K_2 A (\theta - \theta_2)}{l_2}$$

$$\Rightarrow \theta = \frac{\frac{K_1 \theta_1}{l_1} + \frac{K_2 \theta_2}{l_2}}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

Thermal conduction through a cylindrical shell :

Consider a coaxial cylindrical element of radius r and thickness dr . Elements are in series therefore thermal current remains same through all the elements.



$$\frac{dQ}{dt} = \frac{K(2\pi r l) d\theta}{dr} = h$$

$$\Rightarrow h \int_{r_1}^{r_2} \frac{dr}{r} = 2\pi K l \int_{\theta_1}^{\theta_2} d\theta$$

$$\Rightarrow h \ln\left(\frac{r_2}{r_1}\right) = 2\pi K l (\Delta\theta)$$

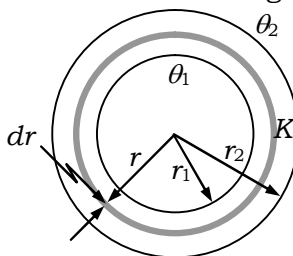
$$\Rightarrow h = \frac{dQ}{dt} = \frac{2\pi K l (\Delta\theta)}{\ln\left(\frac{r_2}{r_1}\right)}$$

in composite cylinder

$$\frac{dQ}{dt} = \frac{2\pi(\Delta\theta) l}{\sum \frac{1}{K_i} \ln\left(\frac{r_{i+1}}{r_i}\right)}$$

Thermal conduction through a spherical shell :

Consider a concentric spherical element of radius r and thickness dr . Elements are in series therefore thermal current remains same through all the elements.



$$\frac{dQ}{dt} = \frac{K(4\pi r^2)d\theta}{dr} = h$$

$$\Rightarrow h \int_{r_1}^{r_2} \frac{dr}{r^2} = 4\pi K \int_{\theta_1}^{\theta_2} d\theta$$

$$\Rightarrow h \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = 4\pi K (\Delta\theta)$$

$$\Rightarrow h = \frac{dQ}{dt} = \frac{4\pi K r_1 r_2 (\Delta\theta)}{(r_2 - r_1)}$$

in composite sphere

$$\frac{dQ}{dt} = \frac{4\pi(\Delta\theta)}{\frac{1}{K_i} \left(\frac{1}{r_i} - \frac{1}{r_{i+1}} \right)}$$

If $(r_2 - r_1)$ is small enough then the formula for cylindrical shell and spherical shell reduces to formula for plane sheet.

Conduction is analogous to current and the analogy is as follows.

$\frac{Q}{t}$	\Leftrightarrow	i	Thermal current (h)
$\Delta\theta$	\Leftrightarrow	V	Temperature difference.
$\frac{d}{KA}$	\Leftrightarrow	R	Thermal resistance (R_t).

Kirchoff's law :

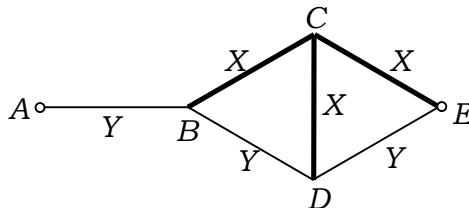
At any node (point where two or more than two conductors meet) the net rate of inflow of heat is always same as the net rate of outflow of heat.

$$\sum h_{in} = \sum h_{out}$$

To apply Kirchoff's law in a thermal circuit, consider one node at a time. Consider currents in all branches connected to that node as incoming to the node. Put the sum of all these currents equal to zero. Repeat this for all the nodes with unknown temperature.

Representative example 2 :

Three rods of material X and three rods of material Y are connected as shown in figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C , calculate the temperatures of the junctions B, C, D. The thermal conductivity of X is $2K$ CGS units and that of Y is K .



Solution :

Let all the rods are of length l and area of cross-section S . Applying kirchoff's current law

At B $\frac{60 - \theta_B}{\left(\frac{l}{KS}\right)} + \frac{\theta_C - \theta_B}{\left(\frac{l}{2KS}\right)} + \frac{\theta_D - \theta_B}{\left(\frac{l}{KS}\right)} = 0$

$\Rightarrow (60 - \theta_B) + 2(\theta_C - \theta_B) + (\theta_D - \theta_B) = 0$

$\Rightarrow -4\theta_B + 2\theta_C + \theta_D = -60 \quad \dots (1)$

At C $\frac{\theta_B - \theta_C}{\left(\frac{l}{2KS}\right)} + \frac{\theta_D - \theta_C}{\left(\frac{l}{2KS}\right)} + \frac{10 - \theta_C}{\left(\frac{l}{2KS}\right)} = 0$

$\Rightarrow (\theta_B - \theta_C) + (\theta_D - \theta_C) + (10 - \theta_C) = 0$

$\Rightarrow \theta_B - 3\theta_C + \theta_D = -10 \quad \dots (2)$

At D $\frac{\theta_B - \theta_D}{\left(\frac{l}{KS}\right)} + \frac{\theta_C - \theta_D}{\left(\frac{l}{2KS}\right)} + \frac{10 - \theta_D}{\left(\frac{l}{KS}\right)} = 0$

$\Rightarrow (\theta_B - \theta_D) + 2(\theta_C - \theta_D) + (10 - \theta_D) = 0$

$\Rightarrow \theta_B + 2\theta_C - 4\theta_D = -10 \quad \dots (3)$

solving (1), (2) and (3) simultaneously we get

$\theta_B = 30^\circ, \theta_C = \theta_D = 20^\circ$

Freezing of a lake :

Let ρ be the density of ice, K be its thermal conductivity and l be the latent heat of gas. Let the temperature above the ice be $-\theta^\circ\text{C}$. When ice first begins to form on a lake, this is because latent heat is absorbed from the surface layer of water by the cold air above it. The subsequent growth of the ice layer requires that the necessary heat abstraction takes place by conduction through the layer of ice already there. Let the initial thickness of ice be x_0 .

At any instant t sec. from the start of freezing, let the thickness of the ice be x . Let the thickness increase by a small amount dx in a small subsequent interval of time of dt .

Let surface area of the lake be A then in a time dt , volume of ice formed

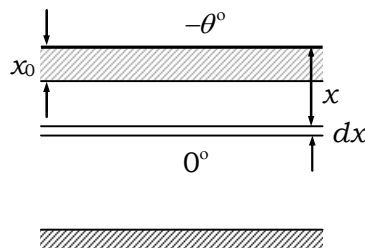
$dV = A dx$

Mass of ice formed

$dm = \rho A dx$

Heat lost to form this ice is

$dQ = \rho A l dx$



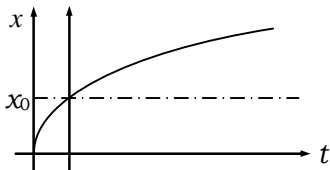
This heat conducted through the layer of ice of thickness x above it in the interval dt to form this ice is

$dQ = \frac{KA\{0 - (-\theta)\} dt}{x} = \rho A l dx$

$\Rightarrow \frac{K\theta}{\rho l} \int_0^t dt = \int_{x_0}^x x dx$

$$\Rightarrow \frac{K\theta}{\rho l} t = \frac{x^2 - x_0^2}{2}$$

$$\Rightarrow x = \sqrt{x_0^2 + 2 \frac{K\theta}{\rho l} t}$$

Convection :

Is a mode of heat transfer in fluids by the mass movement of molecules from one place to another.

The mathematical theory of convection is quite involved. However it has been observed that the ratio at which heat is convected away from an object is approximately

$$\frac{dQ}{dt} \propto A \text{ and } \frac{dQ}{dt} \propto \Delta\theta, \text{ then}$$

$$\frac{dQ}{dt} = h A \Delta\theta$$

Where h is known as convection coefficient.

Radiation :

Is the mode of heat transfer in which medium is not required and the energy moves from source to destination in the form of electromagnetic radiation, radiated by source.

Absorptivity of a surface :

It is defined as the fraction of incident radiation absorbed by the surface. Absorptivity of a surface depends upon the wavelength of incident radiation. That is why different surfaces show different colors corresponding which they have low absorptivity.

Absorptivity of the surface at a particular wavelength a_λ is

$$a_\lambda = \frac{Q_a}{Q_i}$$

$$a_\lambda = \frac{\text{amount of radiation energy of that particular wavelength absorbed by the surface}}{\text{amount of radiation energy of a particular wavelength incident on the same surface}}$$

Perfect Black body :

A perfectly black body is one which completely absorbs the incident radiation energy. It is not necessary that a perfect black body should also look black, instead the color of the black body is the characteristic to its surface temperature.

$$A = 1$$

Reflectivity of a surface :

It is defined as the fraction of incident radiation reflected by the surface. Reflectivity of the surface also depends upon the wavelength of incident radiation.

$$r_\lambda = \frac{Q_r}{Q_i}$$

$$r_\lambda = \frac{\text{amount of radiation energy of that particular wavelength reflected by the surface}}{\text{amount of radiation energy of a particular wavelength incident on the same surface}}$$

Part of incident radiation not absorbed by the surface is reflected back hence

$$r_{\lambda} = 1 - a_{\lambda}$$

reflectivity of the perfect black body is

$$R = 0$$

Emissive power of a surface (e) :

Emissive power of a surface is defined as the total amount of energy radiated by the surface per unit area per unit time.

Stefan's law of radiation (emissive power of perfect black body) :

The emissive power of a surface at a given temperature is proportional to the fourth power of its absolute temperature.

$$e \propto T^4$$

for perfect black body the constant of proportionality is σ , hence

$$E = \sigma T^4 \text{ where } \sigma = 5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$$

Kirchhoff's law :

It states that the ratio of the emissive power to absorptivity of a surface at a particular temperature is a universal constant (is same for all bodies) and is equal to the emissive power of a perfectly black body at same temperature.

$$\left(\frac{e}{a}\right) = E \text{ (constant)}$$

hence from Stefan's law

$$e = aE = a\sigma T^4$$

therefore surfaces with greater absorptivity have greater emissive power, means good absorbers are good emitters.

Emissivity of a surface :

It is defined as the ratio of emissive power of the surface to the emissive power of perfect black body at the same temperature.

$$\varepsilon = \text{Emissivity of the surface} = \frac{\text{emissive power of the surface}}{\text{emissive power of black body at same temperature.}}$$

hence

$$\varepsilon = \frac{e}{E} = a \text{ from Kirchhoff's law.}$$

Newton's law of cooling :

There are two assumptions

1. Both object and environment behave as perfect black bodies.
2. The size of environment is large so that it can provide or absorb any amount of energy without appreciable change in its temperature.

And a restriction

The temperature difference between object and environment is small (less than 20 °C) for Newton's law of cooling to be valid.

If the temperature difference between the object and environment is small, the rate of cooling of the object placed inside a large environment (both behaving like perfect black bodies) is directly proportional to the temperature difference between the object and environment.

According to Stefan's law, the rate of loss of energy from a body at temperature T placed in an environment at temperature T_0 (remains constant) of surface area A is given by

$$\frac{dQ}{dt} = \sigma A (T^4 - T_0^4) = \sigma A ((T_0 + \Delta T)^4 - T_0^4) \text{ (if } \Delta T = (T - T_0) \ll T_0 \text{)}$$

$$\Rightarrow \frac{dQ}{dt} = \sigma A \left\{ T_0^4 \left(1 + 4 \frac{\Delta T}{T_0} \right) - T_0^4 \right\} = 4\sigma A T_0^3 \Delta T$$

But $dQ = -mcdT$ (from the definition of specific heat) therefore

$$-mc \frac{dT}{dt} = 4\sigma A T_0^3 (T - T_0)$$

$$\Rightarrow \frac{dT}{dt} = -\frac{4\sigma A T_0^3}{mc} (T - T_0)$$

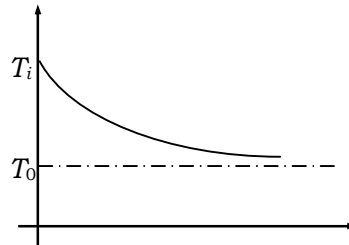
$$\Rightarrow \frac{dT}{dt} \propto -(T - T_0)$$

This proves the Newton's law of cooling. Now

$$\int_{T_i}^{T_f} \frac{dT}{(T - T_0)} = -\left(\frac{4\sigma A T_0^3}{mc} \right) \int_0^t dt$$

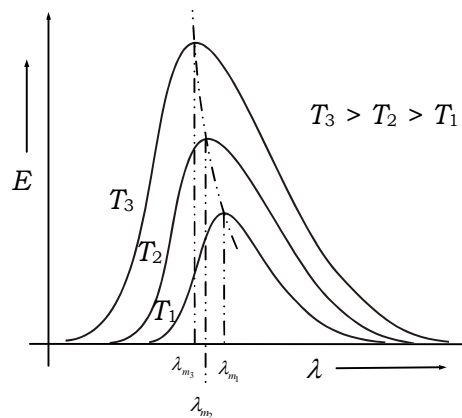
$$\Rightarrow \ln \left(\frac{T - T_0}{T_i - T_0} \right) = -\frac{4\sigma A T_0^3}{mc} t$$

$$\Rightarrow \Delta T = \Delta T_i e^{-\frac{4\sigma A T_0^3}{mc} t}$$



Spectral Distribution of Black - Body Radiation :

Lummer and Pringsheim heated the black body to different temperatures and drew distribution of radiated energy over wavelength for different temperatures on the same graph. The curves so obtained were as shown in the figure. It gives following information.



1. At a given temperature T , with increase in wavelength λ , the energy E_λ first increases, reaches a maximum, and then decreases. This means that for a given temperature, the radiant energy emitted by a black-body is maximum for a particular wavelength (λ_m).

2. As the temperature increases, the peak of the curve shifts towards shorter wavelength i.e. the maximum value of E_λ is obtained at smaller value of λ . Wein established the following relation between temperature T and λ_m ,

$$\lambda_m T = \text{constant.}$$

This is called 'Wein's displacement law'. It shows that as the temperature of the body rises, the maximum energy of the emitted radiation shifts towards lower wavelength.

3. As the temperature rises, the area enclosed by the curve goes on increasing and it verifies the Stefan's law.

4. Wein also proved that at a temperature T , the maximum emitted energy $(E_\lambda)_m$ corresponding to a wavelength λ , is proportional to the fifth power of that temperature (T^5), i.e.

$$(E_\lambda)_m \propto T^5 .$$