HEAT & HEAT TRANSFER

1. A piece of metal floats on mercury. The coefficient of the volume expansion of the metal and the mercury are γ_1 and γ_2 respectively. If the temperature of both mercury and metal are increased by the amount ΔT, what is the factor by which the fraction of the volume of metal submerged in mercury changes?

   Answer : $(\gamma_2 - \gamma_1)\Delta T$

2. What should be lengths of steel and copper rods so that the length of steel rod is 5 cm longer than the copper rod at all temperatures?
   (α_{Cu} for copper = 1.7×10^{-5} per °C and α_s for steel = 1.1×10^{-5} per °C).

   Answer : 14.17 cm , 9.17 cm

3. A certain clock with an iron pendulum is made so as to keep correct time at 10 °C. How will the accuracy of the clock change if the temperature rises to 25 °C?
   Given $\alpha_{iron} = 12 \times 10^{-6}$ per °C.

   Answer : 7.76 sec. per day

4. A glass tube of length 133 cm and of uniform cross-section is to be filled with mercury so that the length of the tube unoccupied by mercury remains same at all temperatures. If cubical coefficients of expansion for glass and mercury are 0.000026 °C^{-1} and 0.000182 °C^{-1}, calculate the length of mercury column.

   Answer : 7.0 cm

5. The coefficient of cubical expansion of mercury is 0.00018 and that of brass 0.00006 per deg. C. If a barometer having a brass-scale were to read 74.5 cm at 30 °C, find the true barometric height at 0 °C. The scale is supposed to be correct at 15 °C.

   Answer : 74.122 cm

6. A weight thermometer contains 500 g of mercury at 0 °C. If the temperature is raised to 100 °C, how much mercury will be expelled. Given cubical expansion of mercury $\gamma_{Hg} = 182 \times 10^{-6}$ °C^{-1} and that of glass $\gamma_g = 27 \times 10^{-6}$ °C^{-1}.

   Answer : 7.63 gm

7. Uniform solid brass cylinder of mass M = 0.50 kg and radius R = 0.030 metre is placed in frictionless bearings and set to rotate about its geometric axis with an angular velocity of 60 radians/sec.
   a) Calculate the angular momentum of cylinder and the work required to reach this state of rotation, staring from rest.
   b) After the cylinder has reached the specified state of rotation, it is heated without any mechanical contact from room temperature 20 °C to 100 °C, find the fractional change in angular velocity of cylinder. Take $\alpha = 2.0 \times 10^{-5}$ per °C.

   Answer : a) 0.0135 J s, 0.4 J ; b) -3.2×10^{-3}

8. Two metal strips, each of length ℓ and thickness d at temperature T_0 are riveted together so that their ends coincide. One strip is made of metal A having a linear coefficient of expansion α_A and other of metal B with a coefficient α_B, where $(\alpha_A < \alpha_B)$. When this bimetallic strip is heated to a temperature ($T_0 + \Delta T$), one strip gets longer than the other hence bimetallic strip bends into an arc of circle. What is the radius of curvature R of the strip?

   Answer : $\dfrac{d}{(\alpha_A - \alpha_B)\Delta T}$

9. A sinker of weight W_0 has an apparent weight W_1 when weighed in a liquid at a temperature T_1 and W_2 when weighed in the same liquid at a temperature T_2. The

coefficient of cubical expansion of the material of sinker is β . What is the coefficient of volume expansion of the liquid?

Answer : $\dfrac{(W_2 - W_1) + \beta(W_0 - W_1)(T_2 - T_1)}{(W_0 - W_2)(T_2 - T_1)}$

10. A vessel is completely filled with 500 g of water and 1000 g of mercury. When 21200 calories of heat are given to it, 3.52 g of water overflows. Calculate the coefficient of volume expansion of mercury. Expansion of vessel may be neglected.
Given, coefficient of volume expansion of water = 1.5×10^{-4} per deg. Density of mercury = 13.6 g/cm^3. Specific heat of mercury = 0.03 cal/g °C.

Answer : 1.768×10^{-4} °C^{-1}

11. A U tube in vertical plane, open at both ends is filled with a liquid. The height of liquid in one arm of tube is ℓ_1 which is maintained at θ_1, and in the other arm it is ℓ_2 which is maintained at θ_2. Find the coefficient of cubical expansion of liquid.

Answer : $\gamma = \dfrac{\ell_1 - \ell_2}{\ell_2\theta_1 - \ell_1\theta_2}$

12. The temperature of equal masses of three different liquids A, B and C are 12 °C, 19 °C and 28 °C respectively. The temperature when A and B are mixed is 16 °C and when B and C are mixed it is 23 °C. What should be the temperature when A and C are mixed?

Answer : 20.26 °C

13. When m gm of substance A at 10 °C is mixed with $2m$ gm of substance B at 30 °C, the temperature of mixture becomes 12 °C. When m gm of substance B at 20 °C is mixed with $2m$ gm of substance C at 50 °C, the temperature of mixture becomes 32 °C. What will be the temperature of mixture of m gm of A at 10 °C, $2m$ gm of B at 20 °C and $3m$ gm of C at 30 °C?

Answer : 11 °C

14. A mixture of 250 g water and 200 g ice at 0 °C is kept in a calorimeter of water equivalent 50 g. If 200 g of steam at 100 °C is passed through this mixture, calculate the final temperature and weight of the contents of the calorimeter. Latent heat of fusion of ice = 80 cal/g. Latent heat of vaporization = 540 gal/g.

Answer : 572.2 gm

15. 20 gm of ice is mixed into 20 gm of water at 80 °C. Find the final temperature of mixture. Latent heat of fusion of ice is 80 cal/gm and specific heat of water is 1 cal/gm °C

Answer : θ = 0 °C

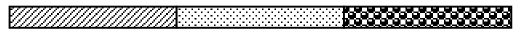
16. A lead ball at 30 °C is dropped from a height of 9.3 km. The ball is heated due to air resistance and it completely melts just before reaching the ground. Calculate the latent heat of fusion of lead. Specific heat capacity of lead = 126 J/kg °C and melting point of lead = 330 °C. Assume that $\left(2/3\right)^{\text{rd}}$ of mechanicals energy lost is used to heat the ball (g = 10 m/s^2).

Answer : 2.42×10^4 J/kg

17. A thermometer has wrong calibration (of course at equal distances and the capillary is of uniform diameter). It reads the melting point of ice as -10 °C. It reads 60 °C in place of 50 °C. What is the temperature of boiling point of water on this scale?

Answer : 130°C

18. Three cylindrical rods A, B, C of equal lengths and equal diameters are joined in series as shown. Their thermal conductivities are $2K$, K, $0.5K$ respectively. At steady state, if the free ends of rods A and C are at 100 °C and 0 °C respectively, calculate the temperature at the two junction points. (Assume negligible loss through the curved surface).What will be the equivalent thermal conductivity of the system?



Answer : 85.7 °C, 57.1 °C, $\frac{2}{7}K$

19. A closed cubical box made of perfectly insulating material has walls of thickness 8 cm and the only way for the heat to enter or leave the box is through two solid, Cylindrical, metallic plugs each of cross-sectional area 12 cm^2 and length 8 cm fixed in the opposite walls of the box. Outer surface A is kept at 100 °C while the outer surface B of the other plug is maintained at 4 °C. The thermal conductivity of the material of the pugs is 0.5 cal s^{-1} cm^{-1} °C^{-1}. A source of energy generating 36 cal s^{-1} is enclosed inside the box. Find the equilibrium temperature of inner surface of the box assuming that it is same at all points on the inner surface.

Answer : 76 °C

20. Water is being boiled in a flat bottom kettle placed on a stove. The area of the bottom is 3000 cm^2 and the thickness is 2 mm. If the amount of steam produced is 1 g/min, calculate the difference of temperature between the inner and outer surfaces of the bottom. The thermal conductivity of material of kettle is 0.5 cal °C^{-1}sec^{-1} and latent heat of steam is 540 cal/g.

Answer : 0.12 °C

21. An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of +20 °C inside it when the outside temperature is –10 °C. The walls have three layers of different material. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 watt m^{-1} °C^{-1} respectively.

Answer : 9 kW

22. A bar of copper of length 75 cm and a bar of Steel of length 125 cm are joined together end to end. Both are of circular cross-section with diameter 2 cm. The free ends of copper and steel are maintained at 100 °C and 0 °C respectively. The surface of the bars are thermally insulated. What is the temperature of copper steel junction? What is the heat transmitted per unit time across the junction?
Thermal conductivity of copper is 9.2×10^{-3} kilo cal/m °C sec and that of steel is 1.1×10^{-3} kilo cal/m °C sec.

Answer : 93.31 °C 26 m cal/s

23. 10 gm of water and 20 gm of ice at 0 °C is kept in a calorimeter of water of equivalent 10 gm. 20 gm of steam at 100 °C is passed in to the calorimeter. Find the final temperature and contents of the calorimeter.

Answer : 100 °C, 40.4 g water and 9.6 g steam.

24. The specific heat of a solid varies according to the law $s = k\theta^2$ where k is a constant and θ is the temperature of the solid. Find the amount of energy required to raise the temperature of m mass of solid from θ_1 to θ_2.

Answer : $\frac{mk}{3}\left(\theta_2^3 - \theta_1^3\right)$

25. A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylindrical block maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 Watt/m-K and the specific heat of the material of the disc is 600 J/kg-K., how long will it take for the temperature of the disc to increase

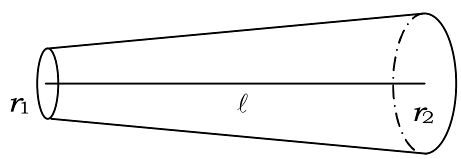
to 350 K? Assume, for purpose of calculation. The thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

Answer : 2 min 46 sec.

26. A large tank of water at constant temperature T_c and a small vessel containing cold water of mass m at initial temperature T_1. A metal rod of length L, area of cross-section A and thermal conductivity K connects the two vessels. Find the time taken for the temperature of water in the smaller vessel to become T_2 ($T_C < T_2 < T_1$). The specific heat capacity of water is c and all other heat capacities are negligible.

Answer : $\frac{mcL}{KA}\log_e\left(\frac{T_c - T_2}{T_c - T_1}\right)$

27. Find the rate of flow of heat through a sloping rod of length ℓ shown in fig. The radii of end faces are r_1 and r_2 respectively. The thermal conductivity of the material of rod is K. The temperatures of ends are θ_1 and θ_2 ($\theta_1 > \theta_2$)



Answer : $\frac{(\theta_1 - \theta_2)K\pi r_1 r_2}{\ell}$

28. Show that the rate of flow of heat through a spherical shell, the inner and outer walls of which have radii r_1 and r_2 and are maintained at different uniform temperatures θ_1 and θ_2 respectively is given by $\frac{dQ}{dt} = \frac{4\pi K r_1 r_2(\theta_1 - \theta_2)}{(r_1 - r_2)}$ Where K is thermal conductivity of material of shell. Hence obtain the expression for temperature distribution.

Answer : $\theta_1 + (\theta_1 - \theta_2)\frac{r_1(r_2 - r)}{r(r_2 - r_1)}$

29. A body of mass 10 g is kept in an enclosure of temperature 27 °C. If the temperature of the body is 127 °C, its specific heat is 0.1 kilo cal/kg °C and area of emitting surface of the body is 10^{-3} m^2, find out the rate of cooling of the body. Stephen's constant $\sigma = 5.72 \times 10^{-8}$ Joule m^{-2} sec^{-1} K^{-4}.

Answer : 0.23 K/sec

30. A body initially at 80 °C cools to 64 °C in 5 minutes and to 52 °C in 10 minutes. What will be the temperature after 15 minutes and what is the temperature of surroundings?

Answer : 43 °C, 16 °C.

31. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no less of heat across the cylindrical surface and the system is in steady state. What is the effective thermal conductivity of the system?

Answer : $\frac{K_1 + 3K_2}{4}$

32. One end of a copper rod of uniform cross-section and of length 1.5 m is kept in contact with ice and he other end with water at 100 °C. At what point along its length should a temperature of 200 °C be maintained so that in steady state, the mass of ice melting be equal to that of the steam produced in the same interval of

time? Assume that the whole system is insulated from the surroundings. Latent heat of fusion of ice and vaporization of water are 80 cal/g and 540 cal/g respectively.

Answer : 1.396 m from ice.

33. Two plates each of area A, thickness L_1 and L_2 and thermal conductivities K_1 and K_2 respectively are joined to form a single plate of thickness $(L_1 + L_2)$. If the temperatures of the free surface are T_1 and T_2, calculate
a) rate of flow of heat
b) temperature of interface and
c) equivalent thermal conductivity.

Answer : a) $\frac{A(T_1 - T_2)K_1K_2}{K_1L_2 + K_2L_1}$; b) $\frac{T_1K_1L_2 + T_2K_2L_1}{K_1L_2 + K_2L_1}$; c) $\frac{(L_1 + L_2)K_1K_2}{K_1L_2 + K_2L_1}$

34. A solid body X of heat capacity C is kept in an atmosphere whose temperature is T_A = 300 K. At time t = 0 the temperature of X is T_0 = 400 K. It cools according to Newton's law of cooling. At time T_1, its temperature is found to be 350 K. At this time (t_1), the body X is connected to a large box Y at atmospheric temperature T_A through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$.

Answer : $300 + 50\exp\left(-2t_1\left[\frac{KA}{CL} + \frac{\log_e 2}{t_1}\right]\right)$

35. A thin metal pipe of 1 meter length and 1 cm radius carries steam at 100 °C. This is covered by two layers of lagging. The thermal conductivity of outer layer, which is 2 cm thick is 3.6×10^{-4} cal/cm- °C-sec while that of inner layer, which is 1 cm thick is 1.2×10^{-4} cal/cm °C sec. If the outer surface of the lagging is at 30 °C, find
a) the temperature of the cylindrical interface of two lagging materials
b) the mass of steam condensed per second. Given $\log_e 2$ = 0.6931.

Answer : a) 47.5 °C; b) 0.0106 gm

36. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectra radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K, calculate
a) the temperature of B and
b) wavelength λ_B.

Answer : a) 1934 K; b) 1.5 μm

37. The time period of a physical pendulum is given by $T = 2\pi\sqrt{\frac{I}{mgl}}$ where m = mass of the pendulum, I = moment of inertia about the axis of suspension, l = distance of cg from the center of suspension. Calculate the change in time period when temperature changes by ΔT. The coefficient of linear expansion of the material of pendulum is α.

Answer : $\frac{1}{2}\alpha t\Delta T$

38. Two rods each of length L_2 and coefficient of linear expansion α_2 each are connected freely to a third rod of length L_1 and coefficient of expansion α_1 to form an isosceles scales triangle. The arrangement is supported on a knife-edge at the midpoint of L_1 which is horizontal. What relation must exist between L_1 and L_2 so that the apex of

the isosceles triangle is to remain at a constant height from the knife edge as the temperature changes?

Answer : $4L_2^2\alpha_2 = L_1^2\alpha_1$

39. The loss of weight of a solid when immersed fn a liquid at 0 °C is W_0. Show that the loss of weight W at t °C is given by

$$W = W_o\{1+(\alpha-\beta)t\},$$

where α and β are the cubical expansion coefficient of solid and liquid respectively.

40. A composite body consists of two rectangular plates of the same dimensions but different thermal conductivities K_A and K_B. This body is used to transfer heat between two objects maintained at different temperatures. The composite body can be placed such that the flow of heat takes place either parallel to the interface or perpendicular to it. Calculate the effective thermal conductivities $K_{||}$ and $K_{\perp}$ of the composite body for the parallel and perpendicular orientations. Which orientation will have more thermal conductivity?

Answer : $\dfrac{K_A+K_B}{2}, \dfrac{2K_AK_B}{K_A+K_B}$