KINETIC THEORY & FIRST LAW OF THERMODYNAMICS

1. Find the mass of air inside a room of dimensions $5 \times 4 \times 3$ m at STP. How much air will come out of the room if the room temperature increases from 15 °C to 27 °C. Atomic weight of air is 28.8.

Answer : 76.2 kg, 2.89 kg

2. A bubble of air when rises from the bottom of a lake to its surface, the radius of the bubble increases from r to $3r$. Considering air to be an ideal gas and neglecting effect of surface tension and change in density of water with depth, find the depth of lake. Atmospheric pressure = 10^5 N/m^2, Density of water assumed independent of depth = 10^3 kg/m^3, acceleration due to gravity = 10 m/s^2.

Answer : 260 m

3. Find the pressure at which helium must be filled in a cylinder of volume 20 lt. so that at 0 °C its weight should be equal to the weight of the open cylinder in atmospheric air at 0 °C. Given atmospheric pressure = 10^5 N/m^2 and molecular weight of air is 28.8 g/mol.

Answer : $p_{He} = 7.2 \times 10^5$ N/m^2

4. A cylinder of volume V = 20 liter is fitted with a valve which opens when pressure inside the cylinder reaches $8P_0$, where $P_0 = 10^5$ N/m^2 is the atmospheric pressure. Now the cylinder is filled with n = 2 moles of Helium gas and heated. At what temperature there will be 1 mole of gas inside the cylinder.

Answer : 1928 K

5. At what temperature rms velocity of Nitrogen molecules will be double the velocity of sound in air at STP. Consider Nitrogen, and air both to be diatomic ideal gas with molecular weight of air 28.8.

Answer : 124 K

6. V_0 volume of an ideal gas at STP is heated isochorically to double its pressure, then it is heated isobarically to double its volume and then allowed to expand isothermally to attain its original pressure from where the isochoric heating started. Find the final temperature and volume of the gas.

Answer : 1092 K, $4V_0$

7. A vertical cylinder of area of cross-section A and height h, is divided in to two parts by a tight fitting, smoothly movable horizontal separator of mass m. Both upper and lower parts of cylinder have one mole of an ideal gas at temperature T_0. Find the ratio of the length of upper and lower parts.

Answer : $\dfrac{mgh}{2RT_0}\left\{1+\sqrt{1+\dfrac{2RT_0}{mgh}}\right\}$

8. An 80 cm long vertical tube, sealed at lower end has a mercury pallet of length 20 cm above 40 cm of air. When the tube is inverted to bring open end down, the mercury pallet touches the open end of the tube. Find the length of trapped air when tube is kept horizontal.

Answer : 48 cm

9. In a tube AB of length 160 cm sealed at both ends has a mercury pallet of length 60 cm trapped in it. When tube is kept horizontal the centre of mercury pallet is at a distance 90 cm from end A. When the tube is tilted at 30° from horizontal with end A lower than B, the centre of mercury pallet is 80 cm from end A. Find the pressure of air in terms height of mercury column inside the tube when it was horizontal.

Answer : 75 cm

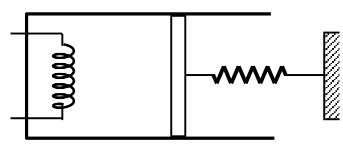
10. An 80 cm high cylindrical vessel with uniform area of cross-section, containing air at atmospheric pressure of 75 cm of mercury column is closed by a light, smooth piston at the top. When mercury is slowly poured on the piston, the air below it compresses isothermally to accommodate it. Find the maximum height to which mercury can stay above piston.

Answer : 5 cm

11. Two communicating glass bulbs of volume V and $2V$ containing ideal gas at STP are connected by a narrow tube. When the bulb of volume V is dipped in boiling water and the other of volume $2V$ is submersed in a liquid maintained at –40 °C find the pressure of gas inside them.

Answer : 1.6×10^5 N/m^2

12. An ideal gas of volume 2400 cm^3 at 300 K and 10^5 N/m^2 is filled in a cylinder of cross-sectional area 80 cm^2. The cylinder is fitted with a movable piston supported by a spring and an electric heater as shown in figure. At the given pressure, temperature and volume of the gas spring is relaxed. Now when gas is heated to 800 K, the piston moves out by 10 cm. Find the force constant of the spring.



Answer : 8000 N/m

13. A vessel of volume V, filled with an ideal gas is being evacuated with the help of a mechanical pump. In each stroke the pump connects a volume ΔV to the vessel and then releases the captured volume of gas to the atmosphere. How many such strokes will be needed to reduce the pressure of gas inside the vessel to half of its original value at constant temperature?

Answer : $n = \dfrac{\ln 2}{\ln\left(1 + \dfrac{\Delta V}{V}\right)}$

14. In a certain process pressure of an ideal gas changes following $p = p_0 - 3V^2$, where V is the molar volume of gas (volume of one mole of gas). What maximum temperature the gas can attain in this process.

Answer : $\dfrac{2p_0^{3/2}}{9R}$

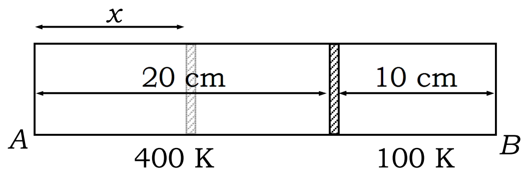
15. In a thermodynamic process temperature of the gas is inversely proportional to its volume. Starting from pressure p_0 and temperature T_0, two moles of gas is cooled to $T_0/2$. Find the final pressure of the gas and work done by gas.

Answer : $p_0/4$; RT_0

16. A cylindrical tube of height 90 cm is divided in to two parts with the help of a smoothly movable separator in the ratio 5 : 4. Both parts contain 0.1 mol of an ideal gas at equal temperature 300 K. Find mass of separator (g = 10 m/s^2).

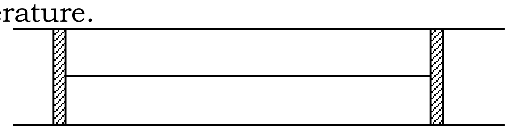
Answer : 12.5 kg

17. A non-conducting cylindrical tube AB of length 30 cm, partitioned by tight fitting, smoothly movable and feebly conducting piston dividing the tube in two parts of length 20 cm and 10 cm respectively as shown in figure. The tube is filled with an ideal gas at 400 K in the left part and at 100 K in the right part at same pressure. When thermal equilibrium between both the parts reached, find the distance of separator from end A.



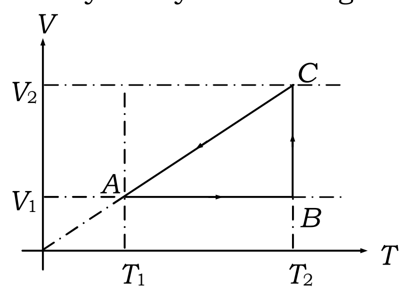
Answer : 10 cm

18. Figure shows a cylindrical tube of cross-sectional area *A*, fitted with two frictionless, non-conducting pistons connected to each other by an inextensible thread. An ideal gas fills the space between the two pistons at temperature T_0 and pressure p_0 (the atmospheric pressure). When the gas is heated, find the tension in the thread as a function of its temperature.



Answer : $\left(1-\frac{T}{T_0}\right)p_0 A$

19. Figure shows a cyclic process *ABCA* performed on *n* moles of an ideal gas. Find the net amount of heat absorbed by the system during complete cycle.



Answer : $nRT_2 \ln\left(V_2/V_1\right) - nR\left(T_2 - T_1\right)$

20. 2 moles of ideal gas at 300 K and 10^5 N/m^2 is heated isochoricaly to increase its absolute temperature to 800 K, and then heated isobaricaly to 1200 K. Find the total heat provided to the gas and work done by gas.

Answer : 29 kJ; 6.65 kJ

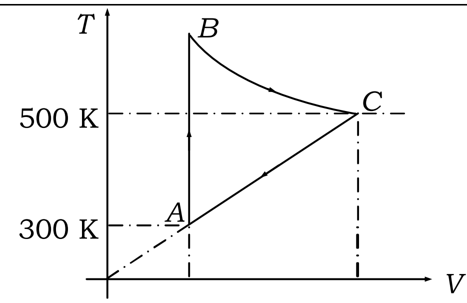
21. Two vessels of volume V_1 and V_2 contain ideal gas at temperatures T_1 and T_2 and pressures p_1 and p_2 respectively. If the gasses are allowed to mix-up by connecting vessels using small tube, find the final temperature and the pressure of the mixture.

Answer : $\frac{p_1V_1 + p_2V_2}{(V_1 + V_2)}$; $\frac{p_1V_1 + p_2V_2}{\frac{p_1V_1}{T_1} + \frac{p_2V_2}{T_2}}$

22. 2 moles of an ideal gas at 300 K is contained in a cylinder of volume 8 litter. Find the work done by it in its isobaric heating to 800 K.

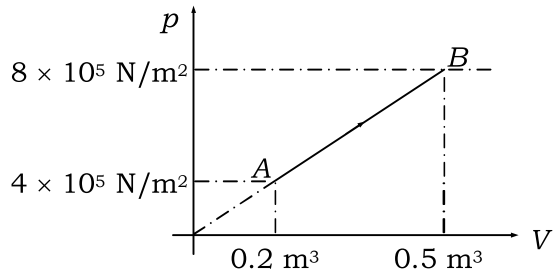
Answer : 8.3 kJ

23. 2 moles of an ideal gas undergoes a cyclic process *ABC* as shown in the following diagram. In the complete cycle total heat absorbed by the gas 1.2 kJ. Find the work done by gas during process from *B* to *C*.



Answer : 4.52 kJ

24. The pressure of a monatomic ideal gas changes linearly with its volume as shown in figure. Find



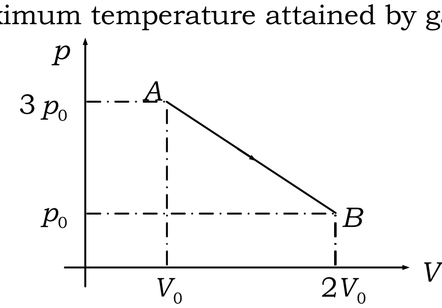
a) work done by gas
b) increase in its internal energy
c) amount of heat absorbed by gas
d) molar heat capacity of the gas.

Answer : a) 180 kJ; b) 480 kJ; c) 660 kJ; d) 17.12 J/mol-K

25. Three moles of an ideal gas at 300 K attains five times its original volume in isothermal expansion and then heated isochoricaly to attain original pressure. In the entire process 83.14 kJ of heat is absorbed by gas. Find whether the gas is monatomic, diatomic or polyatomic, $(\ln 5 = 1.61)$.

Answer : diatomic

26. One mole of an ideal gas is moved from *A* to *B* as shown in the following *p*-*V* diagram. Find the maximum temperature attained by gas in the process.



Answer : $\dfrac{25p_0V_0}{8R}$

27. One mole of an ideal gas with adiabatic exponent γ, follows the thermodynamic process $p = \alpha V$, where α is a constant. The gas expands to increase its volume η times. Find the change in internal energy and heat capacity of the gas.

Answer : $\Delta U = \dfrac{\alpha V^2}{(\gamma - 1)}(\eta^2 - 1)$; $C = \dfrac{R}{2}\left(\dfrac{\gamma + 1}{\gamma - 1}\right)$

28. During a thermodynamic process in an ideal gas with adiabatic exponent γ, one third of heat supplied is used to raise internal energy of gas. Find molar specific heat of the gas in this process.

Answer : $\frac{3R}{(\gamma-1)}$

29. A gaseous mixture enclosed in a vessel consists of 1 g mole of a gas *A* with $\left(\gamma = 5/3\right)$ and another gas *B* with $\left(\gamma = 7/5\right)$ at a temperature *T*. The gas *A* and *B* are mutually inert and can be considered ideal. Find the number of g moles of gas *B* in the mixture if γ of the mixture is $\left(19/13\right)$.

Answer : 2.

30. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27 °C. How much heat has to be transferred to the gas to double the rms speed of its molecules.

Answer : 9340 J

31. An ideal gas at a thermodynamic state (*p*, *V*, *T*) is allowed to expand adiabatically until its volume becomes 5.66*V* while its temperature falls to $\left(T/2\right)$. Find
a) the degree of freedom of gas molecules,
b) the work done by gas during expansion.

Answer : a) $f = 5$; b) $w = \left(5/4\right) pV$

32. A weightless piston divides a thermally insulated cylinder into two parts of volumes *V* = 1 littre and 3*V*. 2 moles of an ideal gas at pressure $p = 2 \times 10^5$ N/m^2 are confined to the part with volume *V*. The remainder of the cylinder is evacuated. Initially the gas is at room temperature (27 °C). The piston is now released and the gas is allowed to expand to fill the entire cylinder. Piston is then pressed back to the initial position. Find the increase of internal energy in the process and final temperature of the gas. Consider γ = 1.5 of the gas.

Answer : 400 J, 312 K

33. 3 moles of a gaseous mixture having volume V and temperature T are compressed to $\left(1/5\right)^{th}$ of its initial volume. Find the change in its adiabatic compressibility, if the gas obeys $pV^{19/13} =$ constant.

Answer : $-0.0248\left(V/T\right)$ m^2/N

34. A gas has molar heat capacity $C = 37.35$ J/mol-K in the thermodynamic process $pT =$ constant. Find the number of degree of freedom of molecules of the gas.

Answer : 5

35. Find the amount work done to increase the temperature of one mole of an ideal gas by 30 °C, if it expands following the process $V \propto T^{2/3}$.

Answer : 166.2 J

36. Find the specific heat of the gas with adiabatic exponent γ, in a process in which work done by gas is same as the increment in internal energy of the gas.

Answer : $\frac{2R}{(\gamma-1)}$

37. Find the specific heat of gas as the function of its volume if one mole of it follows the thermodynamic process $T = T_0 + \alpha p$, where T_0 and α are constants.

Answer : $\frac{\gamma R}{(\gamma-1)} - \frac{V}{\alpha}$

38. Find the equation of the thermodynamic process in which specific heat of the gas is given by $C = \frac{R}{(\gamma - 1)} + \alpha p$.

Answer : $V\left(1 - \alpha \frac{p}{R}\right) =$ constant

39. Find elasticity of an ideal gas in the process pV^2 = constant as a function of its pressure.

Answer : $2p$

40. Find the efficiency of a heat engine in the form of a cyclic process bounded by two isochoric and two isobaric curves, in which ratio of the maximum and minimum volume of gas, as well as the ratio of maximum and minimum pressure of gas is n.

Answer : $\frac{(n-1)(\gamma-1)}{(1+n\gamma)}$

$R = 8.3$ J/mole-K