HEAT & HEAT TRANSFER

HEAT

Thermometry

1. Relation between different scales of temperature

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4}$$

Temperature measurement

$$\theta = \frac{x_\theta - x_0}{(x' - x_0)} \times (\theta' - \theta_0)$$

Temperature coefficient

$$\alpha = \left\{ \frac{x_2 - x_1}{\theta_2 x_1 - \theta_1 x_2} \right\}$$

Calorimetery

Specific heat

$$s = \frac{dQ}{m\, d\theta}$$

Thermal capacity

$$C = \frac{dQ}{d\theta}$$

Water equivalent

$$w = ms = C$$

Latent heat

$$l = \frac{dQ}{dm}$$

Thermal expansion

Coefficient of thermal expansion

$$\alpha = \frac{dl}{l d\theta} \Rightarrow l_\theta = l_0 (1 + \alpha\theta)$$

$$\beta = \frac{dS}{S d\theta} \Rightarrow S_\theta = S_0 (1 + \beta\theta)$$

$$\gamma = \frac{dV}{V d\theta} \Rightarrow V_\theta = V_0 (1 + \gamma\theta)$$

$$\rho_\theta = \frac{\rho_0}{(1 + \gamma\theta)}$$

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

Thermal Stress

$$F/A = -\alpha\, Y\, \Delta\theta$$

Change in time period of simple pendulum

$$\Delta t = \frac{\alpha\, \Delta\theta}{2} t$$

Radius of curvature of bimetallic strip

$$R = \frac{\delta}{(\alpha_1 - \alpha_2)\Delta\theta}$$

HEAT TRANSFER :

Conduction

Rate of flow of heat through a conductor

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{l} = h \text{ (also called thermal current)}$$

Thermal resistance

$$R_T = \frac{\Delta\theta}{h} = \frac{l}{KA}$$

Temperature of interface

$$\theta = \frac{\frac{K_1\theta_1}{l_1} + \frac{K_2\theta_2}{l_2}}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

Rate of flow of heat through cylindrical shell

$$h = \frac{dQ}{dt} = \frac{2\pi K \ell(\Delta\theta)}{\ln\left(r_2/r_1\right)}$$

Rate of flow of heat through spherical shell

$$h = \frac{dQ}{dt} = \frac{4\pi K r_1 r_2(\Delta\theta)}{(r_2 - r_1)}$$

Convection

$\frac{dQ}{dt} \propto A$ and $\frac{dQ}{dt} \propto \Delta\theta$, then

$$\frac{dQ}{dt} = h A \Delta\theta$$

Where h is known as convection coefficient.

Radiation

Absorbitivity of a surface

$$a_\lambda = \frac{\text{amount of radiation energy of that particular wavelength absorbed by the surface}}{\text{amount of radiation energy of a particular wavelength incident on the same surface}}$$

Reflectivity of a surface

$$r_\lambda = \frac{\text{amount of radiation energy of that particular wavelength reflected by the surface}}{\text{amount of radiation energy of a particular wavelength incident on the same surface}}$$

$$r_\lambda = 1 - a_\lambda$$

reflectivity of the perfect black body is

$$R = 0$$

Stefan's law of radiation (emissive power of perfect black body)

$E = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}

Kirchhoff's law

$$\left(\frac{e}{a}\right) = E \text{ (constant)}$$

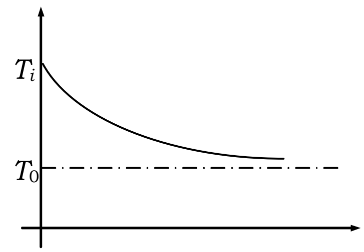
Emissivity of a surface

$$\varepsilon = \textbf{Emissivity of the surface} = \frac{\text{emissive power of the surface}}{\text{emissive power of black body at same temperature.}}$$

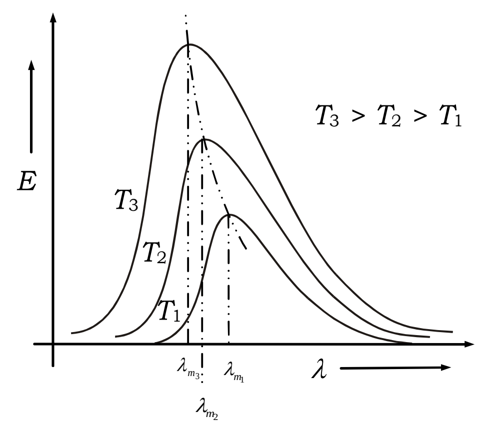
Newton's law of cooling

$$\frac{dQ}{dt} = 4\sigma A T_o^3 \Delta T$$

$$\Rightarrow \quad \Delta T = \Delta T_i e^{-\frac{4\sigma A T_o^3}{mc}t}$$



Spectral Distribution of Black - Body Radiation



$\lambda_m T$ = constant.

GAS LAWS AND KINETIC THEORY OF GASES :

Gas Laws

Boyle's law (Isothermal law)

$$V \propto 1/P \Rightarrow P_1 V_1 = P_2 V_2$$

Charle's Law (Isobaric law)

$$V \propto T \Rightarrow V_1/T_1 = V_2/T_2$$

Pressure Law (Isochoric law)

$$P \propto T \Rightarrow P_1/T_1 = P_2/T_2$$

Ideal Gas Equation

$$PV = nRT \Rightarrow \rho = \frac{PM}{RT}$$

Avogadro's hypothesis

$N = 6.02 \times 10^{23}$ particles per gram-mole

Dalton's law of partial pressure

$$P = \sum p_i \text{ where } p_i = \frac{n_i RT}{V}$$

Grahm's law of diffusion

$$r \propto 1/\sqrt{\rho}$$

Degree of freedom of gas molecules

for monatomic gas

$$f = 3$$

for diatomic or linear poly atomic gas

$$f = 5$$

for non-linear poly atomic gas

$$f = 6$$

relation between γ and f

$$\gamma = 1 + 2/f \Rightarrow f = \frac{2}{(\gamma - 1)}$$

Law of equipartition of energy

$$\langle U_k \rangle = 1/2\, kT$$

$$\Rightarrow \quad U = f/2\, nRT = \frac{nRT}{(\gamma - 1)}$$

Pressure formula for Ideal Gas

pressure of gas

$$P = 1/3\, \rho\, \overline{c^2}$$

Root mean square velocity of gas molecules

$$c_{rms} = \sqrt{\frac{3RT}{M}}$$

mean velocity of gas molecules

$$\bar{c} = \sqrt{\frac{8RT}{\pi M}}$$, and

most probable velocity of gas molecules

$$c_{mp} = \sqrt{\frac{2RT}{M}}$$

Barometric Formula

$$P = P_o e^{-\frac{Mgh}{RT}}$$

Internal Energy of the Ideal Gas

$$U = \frac{f}{2} nRT = \frac{nRT}{(\gamma - 1)}$$

THERMODYNAMICS :

Energies involved in the thermodynamic system

change in internal energy is always

$$\Delta U = \left(f/2\right) nR(\Delta T)$$

Work done by thermodynamic system

$$w = \int_{V_1}^{V_2} PdV$$

Heat exchange with the thermodynamic system

$Q = \Delta U + W$ (First law of thermodynamics)

$$\Rightarrow \quad Q = \frac{f}{2} nR(\Delta T) + \int_{V_1}^{V_2} PdV$$

Thermodynamic Processes

Isochoric Process

$$\Delta U = \frac{f}{2} V(\Delta P)$$

$$w = 0$$

$$Q = \frac{f}{2} V(\Delta P) = \frac{f}{2} nR(\Delta T)$$

Isobaric Process

$$\Delta U = \frac{f}{2} P(\Delta V)$$

$$w = P(\Delta V)$$

$$Q = \left(\frac{f}{2} + 1\right) P(\Delta V) = \left(\frac{f}{2} + 1\right) nR(\Delta T)$$

Isothermal Process

$$\Delta U = 0$$

$$w = nRT \ln\left(V_2/V_1\right) = nRT \ln\left(P_1/P_2\right)$$

$$Q = nRT \ln\left(V_2/V_1\right) = nRT \ln\left(P_1/P_2\right)$$

Adiabatic Process

$$PV^{\gamma} = \text{constant}$$

$$\Rightarrow \quad TV^{(\gamma-1)} = \text{constant}$$

$$\Rightarrow \quad \frac{T^{\gamma}}{P^{(\gamma-1)}} = \text{constant}$$

$$\Delta U = \frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$

$$w = \left(\frac{P_2V_2 - P_1V_1}{1 - \gamma}\right)$$

$$Q = 0$$

Polytropic process

$$\Delta U = \frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$

$$w = \frac{P_2V_2 - P_1V_1}{1 - \eta}$$

$$Q = nR(\Delta T)\left(\frac{1}{\gamma - 1} - \frac{1}{\eta - 1}\right)$$

Specific Heat Of An Ideal Gas

Relation between Mass & molar specific heat

$$C = mc$$

$$C\Big|_{\text{process}} = \frac{dU}{ndT} + \frac{dw}{ndT}\Big|_{\text{process}}$$

$$C\Big|_{\text{process}} = C_V + \frac{PdV}{ndT}\Big|_{\text{process}}$$

$C_P - C_V = R$ called Mayor's result, and

$$C_P/C_V = \gamma$$

Mixture of gases

Average molecular weight

$$\langle M \rangle = \frac{\sum n_i M_i}{\sum n_i}$$

Molar specific heat

$$C_P = \frac{\sum n_i (C_P)_i}{\sum n_i} \text{ and } C_V = \frac{\sum n_i (C_V)_i}{\sum n_i}$$

Adiabatic constant

$$\gamma_{eq} = 1 + \frac{\sum n_i}{\sum \frac{n_i}{(\gamma_i - 1)}} = \frac{\sum n_i C_P}{\sum n_i C_V}$$

Volume elasticity of gases

$$E = \left(\frac{\Delta P}{\Delta V/V} \right) = \frac{V\,\Delta P}{\Delta V}$$

Isothermal elasticity

$$E_{\text{isothermal}} = P_f$$

Adiabatic elasticity

$$E_{\text{adiabatic}} = \gamma P_f$$

Cyclic process

Efficiency of a heat engine operating in a cyclic process

$$\eta = 1 - \frac{Q_{\text{sink}}}{Q_{\text{source}}}$$

Carnot cycle

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$