MATHEMATICAL TOOLS

TRIGONOMETERY

Trigonometric ratios :

S.N.	ratio		0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$(3/2)\pi$	37°	53°
1.	p/h	**sin**	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1	$3/5$	$4/5$
2.	b/h	**cos**	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	–1	0	$4/5$	$3/5$
3.	p/b	**tan**	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	–∞	$3/4$	$4/3$

Basic Identities :

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

A – B Formulae :

1. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
2. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
3. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
4. $\tan\left(A \pm \pi/4\right) = \dfrac{1 \pm \tan A}{1 \mp \tan A}$
5. $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$
6. $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

C – D Formulae :

1. $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
2. $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$
3. $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
4. $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

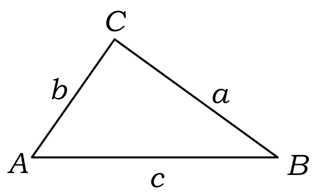
Multiple angle

1. $\sin\theta = 2\sin\left(\theta/2\right)\cos\left(\theta/2\right)$
2. $\cos\theta = \cos^2\left(\theta/2\right) - \sin^2\left(\theta/2\right) = 2\cos^2\left(\theta/2\right) - 1 = 1 - 2\sin^2\left(\theta/2\right)$
3. $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
4. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
5. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
6. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
7. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
8. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$
9. $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$
10. $\cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A\cos B\cos C$

Angle conversion :

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\sin\left(\pi/2 - \theta\right) = \cos\theta$
4. $\cos\left(\pi/2 - \theta\right) = \sin\theta$
5. $\sin\left(\pi/2 + \theta\right) = \cos\theta$
6. $\cos\left(\pi/2 + \theta\right) = -\sin\theta$
7. $\sin(\pi - \theta) = \sin\theta$
8. $\cos(\pi - \theta) = -\cos\theta$
9. $\sin(\pi + \theta) = -\sin\theta$
10. $\cos(\pi + \theta) = -\cos\theta$
11. $\sin(2n\pi \pm \theta) = \pm\sin\theta$
12. $\cos(2n\pi \pm \theta) = \cos\theta$

sine and cosine rule :



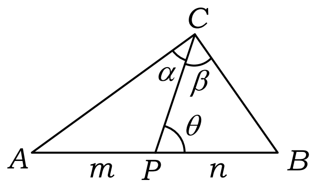
sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Cot m-n theorem :



Applying sin rule in triangles *APC* and *BPC*, and eliminating *A* and *B*

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

Applying sin rule in triangles *APC* and *BPC*, and eliminating α and β

$$(m+n)\cot\theta = n\cot A - m\cot B$$

ALGEBRA :

Factorial :

$n!$ is read as factorial *n*.

$n! = n\times(n-1)!$

$n! = n\times(n-1)\times(n-2)\times\ldots\times3\times2\times1$

$1! = 1$ and $0! = 1$

factorial of any number other than +ve integers is not defined.

Common expansions :

Exponential series :

$$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots \quad = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

logerthemic series :

$$\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+\frac{x^7}{7}-\ldots$$

for $x << 1,\ \ln(1+x)\approx x-\frac{x^2}{2}$

Binomial series :

$$(1+x)^n = 1+\frac{n}{1!}x+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3\ \ldots$$

for $|x|<<1,\ (1+x)^n \approx (1+nx)$

Logerithms :

$\log_a x = y \ \Rightarrow\ x = a^y$

Common notations :

$\log x = \log_{10} x$

$\ln x = \log_e x$

$\lg x = \log_2 x$

Rules of operation :

$\log xy = \log x + \log y$

$\log {}^{x}\!/\!_{y} = \log x - \log y$

$\log x^y = y\log x$

CALCULUS :

DIFFERENTIATION :

Rules of differentiation :

$$\frac{d}{dx}c = 0$$

$$\frac{d}{dx}c\{f(x)\} = c\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\{f_1(x) \pm f_2(x)\} = \frac{d}{dx}f_1(x) \pm \frac{d}{dx}f_2(x)$$

$$\frac{d}{dx}\{f_1(x) \times f_2(x)\} = f_1(x)\frac{d}{dx}f_2(x) + f_2(x)\frac{d}{dx}f_1(x)$$

$$\frac{d}{dx}\left\{\frac{f_1(x)}{f_2(x)}\right\} = \frac{f_2(x)\frac{d}{dx}f_1(x) - f_1(x)\frac{d}{dx}f_2(x)}{f_2^{\,2}(x)}$$

Differentiation formulae :

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\text{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\text{cosec}\, x = -\text{cosec}\, x \cot x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\text{cosec}^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$$

Chain rule :

$$\frac{d}{dx}\{f(x)\}^n = n\{f(x)\}^{n-1}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}e^{f(x)} = e^{f(x)}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}a^{f(x)} = a^{f(x)}\ln a\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\ln\{f(x)\} = \frac{1}{f(x)}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\sin f(x) = \cos f(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\cos f(x) = -\sin f(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\tan f(x) = \sec^2 f(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\cot f(x) = -\text{cosec}^2 f(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\sec f(x) = \sec f(x)\tan f(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\text{cosec} f(x) = -\text{cosec} f(x)\cot f(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\sin^{-1}f(x) = \frac{1}{\sqrt{1-f^2(x)}}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\cos^{-1}f(x) = -\frac{1}{\sqrt{1-f^2(x)}}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\tan^{-1}f(x) = \frac{1}{1+f^2(x)}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\cot^{-1}f(x) = -\frac{1}{1+f^2(x)}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\sec^{-1}f(x) = \frac{1}{f(x)\sqrt{f^2(x)-1}}\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\text{cosec}^{-1}f(x) = -\frac{1}{f(x)\sqrt{f^2(x)-1}}\frac{d}{dx}f(x)$$

DIFFERENTIATION

1. Evaluate the following

1. $\frac{d}{dx}(3\sin x+5\tan x)$
2. $\frac{d}{dx}(4+3\tan^{-1}x)$
3. $\frac{d}{dx}(4x+3x^4+\ln x)$
4. $\frac{d}{dx}(\sin^{-1}x+4\sec x)$
5. $\frac{d}{dx}(6e^x+5^x)$
6. $\frac{d}{dx}\left(x^{1/3}+\sec^{-1}x\right)$
7. $\frac{d}{dx}\left(\frac{1}{x}+\tan x+x^2+\ln x\right)$
8. $\frac{d}{dx}(4\cos x+\cot x-3e^x)$
9. $\frac{d}{dx}\left(x^{-7/2}+x^{2/3}+\operatorname{cosec} x\right)$
10. $\frac{d}{dx}(\sin x.\ln x)$
11. $\frac{d}{dx}(3e^x.\tan x)$
12. $\frac{d}{dx}(\ln x+\tan x)$
13. $\frac{d}{dx}(3\sin x+e^x).\ln x$
14. $\frac{d}{dx}\tan x.(\ln x+4^x)$
15. $\frac{d}{dx}(\sin x.e^x.x^4)$
16. $\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$
17. $\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$
18. $\frac{d}{dx}\left(\frac{1}{\cos x}\right)$
19. $\frac{d}{dx}\left(\frac{1}{\sin x}\right)$
20. $\frac{d}{dx}\left(\frac{\tan x+\tan^{-1}x}{x^4}\right)$
21. $\frac{d}{dx}\frac{(e^x+\ln x)\sin x}{(x^2+2\tan x)}$

2. Evaluate the following

1. $\frac{d}{dx}\sin^3 x$
2. $\frac{d}{dx}\tan x^3$
3. $\frac{d}{dx}e^{\sin x}$
4. $\frac{d}{dx}\sin(\ln x)$
5. $\frac{d}{dx}\ln(\tan x)+e^{x^3}+3\tan^2 x$
6. $\frac{d}{dx}(\ln x)^5$
7. $\frac{d}{dx}\left(\sin^5 x+\frac{5\tan x}{\ln x}\right)$
8. $\frac{d}{dx}\left\{\sec(e^x)+\sin^{-1}x^2\right\}$
9. $\frac{d}{dx}\left(e^{\tan^{-1}x}\right)$
10. $\frac{d}{dx}\ln(\sin(e^x))$
11. $\frac{d}{dx}\sec^{-1}(\ln x)$
12. $\frac{d}{dx}\left(\tan^{-1}(\sin x)+\sin^{-1}(\tan x)\right)$
13. $\frac{d}{dx}\ln(x^6.\sin(e^x))$
14. $\frac{d}{dx}\left(e^3+\sin(x^3)+\ln(4+3\sin x)\right)$

3. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

1. $y = x^4 + \sin x$
2. $y = x^2 + x + \frac{1}{x}$
3. $y = x \ln x$
4. $y = \tan x + e^x$
5. $y = e^x \sin x$
6. $y = \sin x + e^x + \tan^{-1} x$
7. $y = x^4 \sin x$
8. $y = 4 \ln x + x^6 + \sec x$

4. Implicitly differentiate and find $\frac{dy}{dx}$

1. $x^2 + y^2 = 1$
2. $y^2 = \frac{x-1}{x+1}$
3. $x^2 y + xy^2 = 6$
4. $y^2 = x^3$
5. $2xy + y^2 = x + y$
6. $x^2 y^2 = x^2 + y^2$
7. $y = x^2 + \frac{1}{y^2}$
8. $x \sin y + y \sin x = \log(xy)$
9. $x^2 + \tan x + \sin y \log x + xy^2 = 6$
10. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants.

5. Find $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$

1. $V = A(xy + yz + zx)$
2. $V = 10x^2 y + 8xyz + 2x^2 \sin y$
3. $V = x^2 + y^2 + xyz$
4. $V = x^2 yz + xyz^2 + x + y$

ANSWER

1.
1. $3\cos x + 5\sec^2 x$
2. $\dfrac{3}{1+x^2}$
3. $4+12x^3+\dfrac{1}{x}$
4. $\dfrac{1}{\sqrt{1-x^2}}+4\sec x\tan x$
5. $6e^x+5^x\ln 5$
6. $\dfrac{1}{3}x^{-2/3}+\dfrac{1}{x\sqrt{x^2-1}}$
7. $-\dfrac{1}{x^2}+\sec^2 x+2x+\dfrac{1}{x}$
8. $-\left(4\sin x+\operatorname{cosec}^2 x+3e^x\right)$
9. $-\dfrac{7}{2}x^{-9/2}+\dfrac{2}{3}x^{-1/3}-\operatorname{cosec}x\cot x$
10. $\dfrac{\sin x}{x}+\cos x\ln x$
11. $3e^x\left(\sec^2 x+\tan x\right)$
12. $\dfrac{1}{x}+\sec^2 x$
13. $\left(3\cos x+e^x\right)\ln x+\dfrac{1}{x}\left(3\sin x+e^x\right)$
14. $\tan x\left(\dfrac{1}{x}+4^x\ln 4\right)+\left(\ln x+4^x\right)\sec^2 x$
15. $x^4e^x\left(\sin x\ln 4+\sin x+\cos x\right)$
16. $\sec^2 x$
17. $-\operatorname{cosec}^2 x$
18. $\sec x\tan x$
19. $-\operatorname{cosec}x\cot x$
20. $\dfrac{1}{x^4}\left(\sec^2 x+\dfrac{1}{\left(1+x^2\right)}\right)-\dfrac{4}{x^5}\left(\tan x+\tan^{-1}x\right)$
21. $\dfrac{\left(x^2+2\tan x\right)\left\{\left(e^x+\ln x\right)\cos x+\sin x\left(e^x+1/x\right)\right\}-\left(e^x+\ln x\right)\sin x\left(2x+2\sec^2 x\right)}{\left(x^2+2\tan x\right)^2}$

2.
1. $3\sin^2 x\cos x$
2. $\left(\sec^2 x^3\right)3x^2$
3. $\left(e^{\sin x}\right)\cos x$
4. $\dfrac{\cos(\ln x)}{x}$
5. $\dfrac{2}{\sin 2x}+3x^2e^{x^3}+6\tan x\sec^2 x$
6. $\dfrac{5(\ln x)^4}{x}$
7. $5\left(\sin^4 x\cos x+\dfrac{\ln x\sec^2 x-\dfrac{\tan x}{x}}{(\ln x)^2}\right)$
8. $\sec\left(e^x\right)\tan\left(e^x\right)e^x+\dfrac{2x}{\sqrt{1-x^4}}$
9. $e^{\tan^{-1}x}\dfrac{1}{\left(1+x^2\right)}$
10. $\cot\left(e^x\right)e^x$
11. $\dfrac{1}{x\ln x\sqrt{\ln^2 x-1}}$
12. $\dfrac{\cos x}{1+\sin^2 x}+\dfrac{\sec^2 x}{\sqrt{1-\tan^2 x}}$
13. $\dfrac{x\cos\left(e^x\right)e^x+6\sin\left(e^x\right)}{\sin\left(e^x\right)}$
14. $\cos\left(x^3\right).3x^2+\dfrac{3\cos x}{4+3\sin x}$

3.
1. $4x^3+\cos x,\quad 12x^2-\sin x$
2. $2x+1-\dfrac{1}{x^2},\quad 2\left(1+\dfrac{1}{x^3}\right)$
3. $1+\ln x,\quad \dfrac{1}{x}$
4. $\sec^2 x+e^x,\quad 2\sec^2 x\tan x+e^x$
5. $e^x\left(\cos x+\sin x\right),\quad 2e^x\cos x$

6. $\cos x + e^x + \frac{1}{1+x^2}$, $-\sin x + e^x - \frac{2x}{\left(1+x^2\right)^2}$

7. $x^4 \cos x + 4x^3 \sin x$, $-x^4 \sin x + 8x^3 \cos x + 12x^2 \sin x$

8. $\frac{4}{x} + 6x^5 + \sec x \tan x$, $-\frac{4}{x^2} + 30x^4 + \sec^3 x\left(1+\sin^2 x\right)$

4. 1. $-x/y$

2. $\frac{1}{y(x+1)}$

3. $-\frac{(2x+y)y}{(x+2y)x}$

4. $\frac{3x^2}{2y}$

5. $\frac{1-2y}{2x+2y-1}$

6. $= \frac{x\left(1-y^2\right)}{\left(x^2-1\right)y}$

7. $\frac{2xy^3}{y^3+2}$

8. $\frac{1-x(\sin y + y\cos x)}{y(x\cos y + \sin x)-1}$

9. $= -\frac{x\left(y^2+2x+\sec^2 x\right)+\sin y}{x(\ln x \cos y + 2xy)}$

10. $-\frac{ax+hy+g}{hx+by+f}$

5. 1. $A(y+z)$, $A(x+z)$, $A(x+y)$

2. $20xy + 8yz + 4x\sin y$, $10x^2 + 8xz + 2x^2\cos y$, $V = 8xy$

3. $(2x+yz)$, $(2y+xz)$, xy

4. $2xyz + yz^2 + 1$, $x^2z + xz^2 + 1$, $x^2y + 2xyz$

INTEGRATION :

Rules of integration :

$$\int dx = x + k$$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int \{f_1(x) \pm f_2(x)\}dx = \int f_1(x)dx \pm \int f_2(x)dx$$

$$\int f_1(x).f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$$

$$\int \frac{f'(x)dx}{f(x)} = \ln f(x) + c \text{ where } f'(x) = \frac{d}{dx}f(x)$$

A brief list of standard integrals :

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + k$

2. $\int xdy + \int ydx = xy + k$

3. $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + k$ where $f'(x) = \frac{d}{dx}f(x)$.

4. $\int e^x dx = e^x + k$

5. $\int a^x dx = \frac{a^x}{\ln a} + k$

6. $\int \cos xdx = \sin x + k$

7. $\int \sin xdx = -\cos x + k$

8. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + k$

9. $\int \frac{dx}{(ax+b)} = \frac{1}{a}\ln|ax+b| + k$

10. $\int x(ax+b)^n dx = \frac{(ax+b)^{n+1}}{a^2}\left\{\frac{ax+b}{n+2} - \frac{b}{n+1}\right\} + k$

11. $\int \frac{xdx}{(ax+b)} = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + k$

12. $\int \frac{xdx}{(ax+b)^2} = \frac{1}{a^2}\left\{\ln|ax+b| + \frac{b}{ax+b}\right\} + k$

13. $\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + k$

14. $\int (ax+b)^{n/2} dx = \frac{2}{a}\frac{(ax+b)^{n/2+1}}{n+2} + k$

15. $\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$

16. $\int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}}\tan^{-1}\sqrt{\frac{ax-b}{b}} + k$

17. $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + k$

18. $\int \frac{\sqrt{ax+b}}{x^2}dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int\frac{dx}{x\sqrt{ax+b}} + k$

19. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int\frac{dx}{x\sqrt{ax+b}} + k$

20. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + k$

21. $\int \frac{dx}{\left(a^2+x^2\right)^2} = \frac{x}{2a^2\left(a^2+x^2\right)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) + k$

22. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + k$

23. $\int \frac{dx}{\left(a^2-x^2\right)^2} = \frac{x}{2a^2\left(a^2-x^2\right)} + \frac{1}{2a^2}\int\frac{dx}{a^2-x^2}$

24. $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right| + k$

25. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + k$

26. $\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}\left(a^2+2x^2\right)\sqrt{a^2+x^2} - \frac{a^4}{8}\ln\left|x+\sqrt{a^2+x^2}\right| + k$

27. $\int \frac{\sqrt{a^2+x^2}}{x}dx = \sqrt{a^2+x^2} - a\ln\left|a+\sqrt{a^2+x^2}\right| + k$

28. $\int \frac{\sqrt{a^2+x^2}}{x^2}dx = \ln\left|x+\sqrt{a^2+x^2}\right| - \frac{\sqrt{a^2+x^2}}{x} + k$

29. $\int \frac{x^2}{\sqrt{a^2+x^2}}dx = -\frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + \frac{x\sqrt{a^2+x^2}}{2} + k$

30. $\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + k$

31. $\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + k$

32. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + k$

33. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + k$

34. $\int x^2\sqrt{a^2-x^2}\,dx = \frac{a^4}{8}\sin^{-1}\left(\frac{x}{a}\right) - \frac{1}{8}x\sqrt{a^2-x^2}\left(a^2-2x^2\right) + k$

35. $\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + k$

36. $\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\sin^{-1}\left(\frac{x}{a}\right) - \frac{\sqrt{a^2-x^2}}{x} + k$

37. $\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)-\frac{1}{2}x\sqrt{a^2-x^2}+k$

38. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|+k$

39. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}+k$

40. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right|+c$

41. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+k$

42. $\int \left(x^2-a^2\right)^{n/2}dx = \frac{x\left(x^2-a^2\right)^{n/2}}{n+1}-\frac{na^2}{n+1}\int\left(x^2-a^2\right)^{n/2-1}dx$

43. $\int \frac{dx}{\left(x^2-a^2\right)^{n/2}} = \frac{x\left(x^2-a^2\right)^{1-n/2}}{(2-n)a^2}-\frac{n-3}{(n-2)a^2}\int\frac{dx}{\left(x^2-a^2\right)^{n/2-1}}$

44. $\int x\left(x^2-a^2\right)^{n/2}dx = \frac{\left(x^2-a^2\right)^{n/2+1}}{n+2}+k$

45. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x}{8}\left(2x^2-a^2\right)\sqrt{x^2-a^2}-\frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right|+k$

46. $\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2}-a\sec^{-1}\left|\frac{x}{a}\right|+k$

47. $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = \ln\left|x+\sqrt{x^2-a^2}\right|-\frac{\sqrt{x^2-a^2}}{x}+k$

48. $\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx = \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+\frac{x}{2}\sqrt{x^2-a^2}+k$

49. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|+k$

50. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+k$

51. $\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right)+k$

52. $\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+k$

53. $\int \left(2ax-x^2\right)^{n/2}dx = \frac{(x-a)\left(2ax-x^2\right)^{n/2}}{n+1}+\frac{na^2}{n+1}\int\left(2ax-x^2\right)^{n/2-1}dx$

54. $\int \frac{dx}{\left(2ax-x^2\right)^{n/2}} = \frac{(x-a)\left(2ax-x^2\right)^{1-n/2}}{(n-2)a^2}+\frac{(n-3)a^2}{(n-2)a^2}\int\frac{dx}{\left(2ax-x^2\right)^{n/2-1}}$

55. $\int x\sqrt{2ax-x^2}\,dx = \frac{(x+a)(2x-3a)}{6}\sqrt{2ax-x^2}+\frac{a^3}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+k$

56. $\int \frac{\sqrt{2ax-x^2}}{x}\,dx = \sqrt{2ax-x^2}+a\sin^{-1}\left(\frac{x-a}{a}\right)+k$

57. $\int \frac{\sqrt{2ax-x^2}}{x^2}dx = -2\sqrt{\frac{2a-x}{x}} - \sin^{-1}\left(\frac{x-a}{a}\right) + k$

58. $\int \frac{xdx}{\sqrt{2ax-x^2}} = a\sin^{-1}\left(\frac{x-a}{a}\right) - \sqrt{2ax-x^2} + k$

59. $\int \frac{dx}{x\sqrt{2ax-x^2}} = -\frac{1}{a}\sqrt{\frac{2a-x}{x}} + k$

60. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + k$

61. $\int \cos ax\,dx = \frac{1}{a}\sin ax + k$

62. $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + k$

63. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + k$

64. $\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax\cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} axdx$

65. $\int \cos^n ax\,dx = -\frac{\sin^{n-1} ax\cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} axdx$

66. $\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \sin^n x\,dx = \begin{cases} \frac{1.3.5....(n-1)}{2.4.6....n}.\frac{\pi}{2} & \text{if } n \text{ is even} \geq 2 \\ \frac{2.4.6....(n-1)}{3.5.7....n} & \text{if } n \text{ is odd } \geq 3 \end{cases}$

67. $\int \sin ax\cos bxdx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + k$

68. $\int \sin ax\sin bxdx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + k$

69. $\int \cos ax\cos bxdx = -\frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + k$

70. $\int \sin^n ax\cos axdx = \frac{\sin^{n+1} ax}{(n+1)a} + k$

71. $\int \tan ax\,dx = \frac{1}{a}\ln|\sec ax| + k$

72. $\int \cot ax\,dx = \frac{1}{a}\ln|\sin ax| + k$

73. $\int \cos^n ax\sin ax\,dx = -\frac{\cos^{n+1} ax}{(n+1)a} + k$

74. $\int \sin^n ax\cos^m ax\,dx = -\frac{\sin^{n-1} ax\cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n}\int \sin^{n-2} ax\cos^m ax\,dx$

$= -\frac{\sin^{n+1} ax\cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n}\int \sin^n ax\cos^{m-2} ax\,dx$

75. $\int \frac{dx}{b+c\sin ax} = -\frac{2}{a\sqrt{b^2-c^2}}\tan^{-1}\left\{\sqrt{\frac{b-c}{b+c}}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right\} + k \quad \text{if } b^2 > c^2$

$= -\frac{1}{a\sqrt{c^2-b^2}}\ln\left|\frac{c+b\sin ax+\sqrt{c^2-b^2}\cos ax}{b+c\sin ax}\right|+k \text{ if } b^2<c^2$

76. $\int\frac{dx}{1+\sin ax} = -\frac{1}{a}\tan\left(\pi/4-\frac{ax}{2}\right)+k$

77. $\int\frac{dx}{1-\sin ax} = \frac{1}{a}\tan\left(\pi/4+\frac{ax}{2}\right)+k$

78. $\int\frac{dx}{b+c\cos ax} = \frac{2}{a\sqrt{b^2-c^2}}\tan^{-1}\left\{\sqrt{\frac{b-c}{b+c}}\tan\left(\frac{ax}{2}\right)\right\}+k \quad \text{if } b^2>c^2$

$= \frac{1}{a\sqrt{c^2-b^2}}\ln\left|\frac{c+b\cos ax+\sqrt{c^2-b^2}\sin ax}{b+c\cos ax}\right|+k \quad \text{if } b^2<c^2$

79. $\int\frac{dx}{1+\cos ax} = \frac{1}{a}\tan\left(\frac{ax}{2}\right)+k$

80. $\int\frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\left(\frac{ax}{2}\right)+k$

81. $\int x\sin ax dx = \frac{1}{a^2}\sin ax-\frac{x}{a}\cos ax+k$

82. $\int x\sin ax dx = \frac{1}{a^2}\sin ax-\frac{x}{a}\cos ax+k$

83. $\int x\sin ax dx = -\frac{x^n}{a}\cos ax+\frac{n}{a}\int x^{n-1}\cos ax dx$

84. $\int x^n\cos ax dx = -\frac{x^n}{a}\cos ax+\frac{n}{a}\int x^{n-1}\cos ax dx$

85. $\int\tan ax dx = -\frac{1}{a}\ln|\cos ax|+k$

86. $\int\tan ax dx = \frac{1}{a}\ln|\sin ax|+k$

87. $\int\tan^2 ax\,dx = \frac{1}{a}\tan ax-x+k$

88. $\int\cot^2 ax\,dx = -\frac{1}{a}\cot ax-x+k$

89. $\int\tan^n ax\,dx = \frac{\tan^{n-1}ax}{a(n-1)}-\int\tan^{n-2}ax\,dx$

90. $\int\cot^n ax\,dx = -\frac{\cot^{n-1}ax}{a(n-1)}-\int\cot^{n-2}ax\,dx$

91. $\int\sec ax\,dx = \frac{1}{a}\ln|\sec ax+\tan ax|+k = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|+k$

92. $\int\text{cosec}\,ax\,dx = -\frac{1}{a}\ln|\text{cosec}\,ax+\cot ax|+k = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2}\right)\right|+k$

93. $\int\sec^2 ax\,dx = \frac{1}{a}\tan ax+k$

94. $\int\text{cosec}^2 ax\,dx = -\frac{1}{a}\cot ax+k$

95. $\int \sec^n ax\,dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} ax + k$

96. $\int \text{cosec}^n ax\,dx = -\frac{\text{cosec}^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1}\int \text{cosec}^{n-2} ax\,dx + k$

97. $\int \sec^n ax \tan ax\,dx = \frac{\sec^n ax}{na} + k$

98. $\int \text{cosec}^n ax \cot ax\,dx = -\frac{\text{cosec}^n ax}{na} + k$

99. $\int \sin^{-1} ax dx = x\sin^{-1} ax + \frac{1}{a}\sqrt{1-a^2x^2} + k$

100. $\int \cos^{-1} ax dx = x\cos^{-1} ax - \frac{1}{a}\sqrt{1-a^2x^2} + k$

101. $\int \tan^{-1} ax dx = x\tan^{-1} ax - \frac{1}{2a}\ln\left(1+a^2x^2\right) + k$

102. $\int e^{ax} dx = \frac{e^{ax}}{a} + k$

103. $\int b^{ax} dx = \frac{b^{ax}}{a\ln b} + k$

104. $\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax-1) + k$

105. $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax} dx + k$

106. $\int x^n b^{ax} dx = \frac{x^n b^{ax}}{a\ln b} - \frac{n}{a\ln b}\int x^{n-1} b^{ax} dx + k$

107. $\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + k$

108. $\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + k$

109 $\int_0^\infty \frac{x^{n-1}}{e^x} dx = (n-1)!$

110. $\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

111. $\int \ln ax dx = x\ln ax - x + k$

112. $\int x^n \ln ax dx = \frac{x^{n+1}}{n+1}\ln ax - \frac{x^{n+1}}{(n+1)^2} + k$

113. $\int \frac{\ln ax}{x} dx = \frac{(\ln ax)^2}{2} + k$

114. $\int \frac{dx}{x\ln x} = \ln\left|\ln ax\right| + k$

INTEGRATION

1. If the acceleration of the particle varies by following relations then find the expression for the velocity of the particle as a function of time. Given initial velocity of the particle is zero.

1. $a = t^2 - \frac{1}{t^2} + 2t^3 + 12$
2. $a = e^{(t+2)}$
3. $a = \frac{1}{t} + \frac{1}{1+t}$
4. $a = t^{3/2} + \frac{1}{t^{1/2}} - 11$
5. $a = 4t^7 + 3t^2 + t - 5$

2. If the velocity of particle varies by following relations then find the expression for the displacement of the particle

1. $v = \frac{1}{\sqrt{9t^2 - 4}}$
2. $v = \frac{t}{\sqrt{t+2}}$
3. $v = t\sqrt{t+3}$
4. $v = \frac{1}{\sqrt[3]{t^2}}$

3. A particle travels along a straight line with acceleration varying as $a = t + 2$ If the velocity of particle is 32 ms^{-1} when time elapsed is 4 sec. Find the expression for displacement of the particle.

4. Calculate the following

1. $\int (1-x)\sqrt{x}\; dx$
2. $\int (2-3x)(3+2x)(1-2x)\, dx$
3. $\int \tan 2x\, dx$
4. $\int \frac{1}{\sin^2 x \cos^2 x} dx$
5. $\int \frac{1}{1+\cos x} dx$
6. $\int \frac{\sin x}{1+\sin x} dx$
7. $\int \sin^{-1}(\cos x)\, dx$
8. $\int \frac{(a^x + b^x)^2}{a^x b^x} dx$
9. $\int \frac{x+2}{(x+1)^2} dx$
10. $\int \left(6\sin x \cos x + 4\sec 2x\ \text{cosec} 2x\right) dx$

5. Evaluate the following

1. $\int_{-4}^{-1} \frac{1}{x} dx$
2. $\int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}} dx$
3. $\int_0^1 (3x^2 + 2x - 2)\, dx$
4. $\int_0^1 xe^x\, dx$
5. $\int_0^{\pi/2} x \sin x\, dx$
6. $\int_0^1 xe^x + \sin\frac{\pi x}{4} dx$
7. $\int_{\pi/4}^{\pi/2} \cot x dx$
8. $\int_0^1 \frac{1-x}{1+x} dx$
9. $\int_0^\infty e^{-x} dx$
10. $\int_0^\pi \frac{1}{1+\sin x} dx$

6. Find using integration,

i) the area of a disk of radius R
ii) the volume of sphere of radius R
iii) the surface area of spherical shell of radius R
iv) the volume of cone of base radius R and height h

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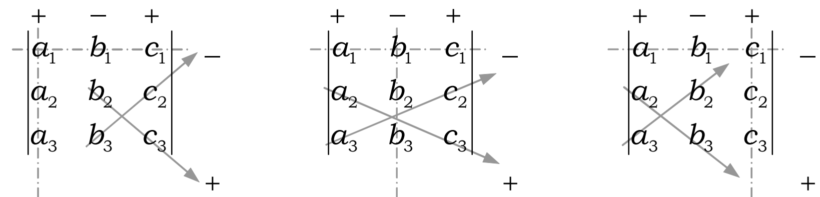
DETERMINANTS :

A mathematical object of the form

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

is defined as the determinant of order 3. To evaluate the determinant use top row or left column (any row or column can be used but it is easier to use top row or left column). Let us use top row. Mark the elements with alternate +ve and –ve sign of top row starting with +ve sign. Open the structure with one element of top row at a time. The row or column of which the element is the member should be hide. Rest four elements will be left. They are multiplied as shown and downward diagonal is taken positive where as upward diagonal is taken negative.



$+a_1(b_2c_3 - b_3c_2)$, $-b_1(a_2c_3 - a_3c_2)$, $+c_1(a_2b_3 - a_3b_2)$

hence

$$\Delta = a_1\left(b_2c_3 - b_3c_2\right) - b_1\left(a_2c_3 - a_3c_2\right) + c_1\left(a_2b_3 - a_3b_2\right)$$

VECTORS :

Physical quantities are broadly in to four categories.

1. Scaler :
The physical quantities having magnitude only and having no sense of direction are called scalar quantities.

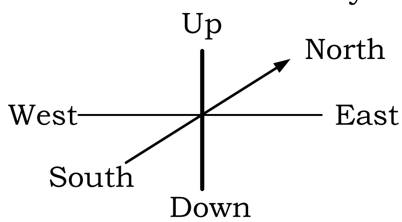
2. Vector :
The physical quantities having magnitude as well as sense of absolute direction are called vector quantities.
Vectors follow different set of mathematical operations defined in vector algebra.
Symbolically they are represented as

$\mathbf{a}$, $\mathbf{A}$, $\bar{a}$, $\vec{a}$, $\underline{a}$ or $\vec{\mathbf{a}}$

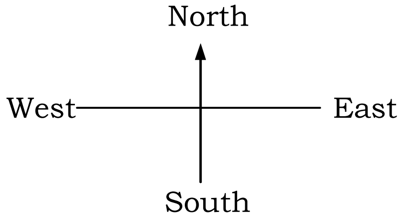
and their magnitude is represented as

$|\vec{a}|$, $|\vec{\mathbf{a}}|$, $|\mathbf{a}|$ or a

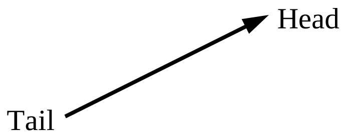
For a three dimensional system



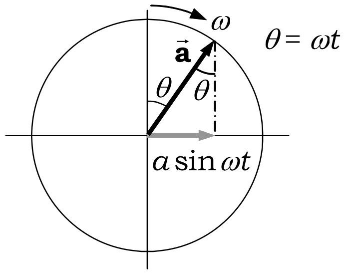
For a two dimensional system



Graphically they are represented by a line segment with an arrow at the end, oriented in the direction of vector. The length of the line segment is proportional to the magnitude of vector quantity.



3. Phaser :
The physical quantities whose magnitude can be represented as the magnitude of projection of a rotating vector are called phaser quantities. These quantities do not have sense of absolute direction. The angle of rotation of the rotating vector is called phase angle, which decides the magnitude of the phaser quantity.
In SHM displacement, velocity and acceleration of the oscillating object are phaser quantities. In sinusoidal alternating current the current and voltages are phaser quantities.



4. Tenser :
These are beyond the scope of our discussion.

<u>Unit vector</u> :
A vector having unit magnitude is called a unit vector. It is represented by

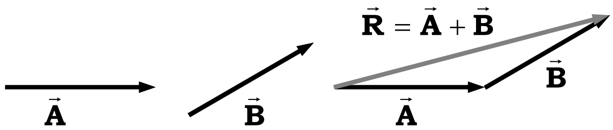
$\hat{\mathbf{a}}$ or $\hat{a}$

<u>Null vector</u> :
A vector with zero magnitude is called a null vector. When a vector is having zero magnitude its direction can be arbitrarily taken. It is represented by

$\vec{\mathbf{0}}$ or $\vec{0}$

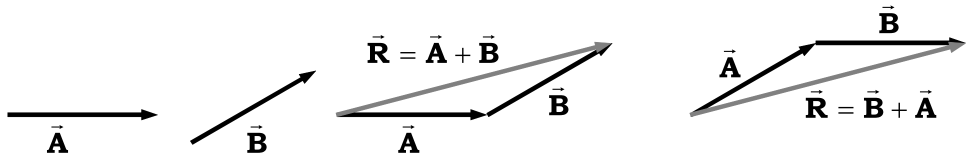
<u>Operations on vector quantities</u> :

<u>Addition</u> :
To add two vectors, first draw a vector and then draw other vector joining tail of the other vector with head of the first vector. The resultant of the two vectors is then found by joining tail of the first vector with the head of the second vector as shown.



Vector addition is commutative :

Adding vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$ gives same result when we add vector $\vec{\mathbf{A}}$ to vector $\vec{\mathbf{B}}$.



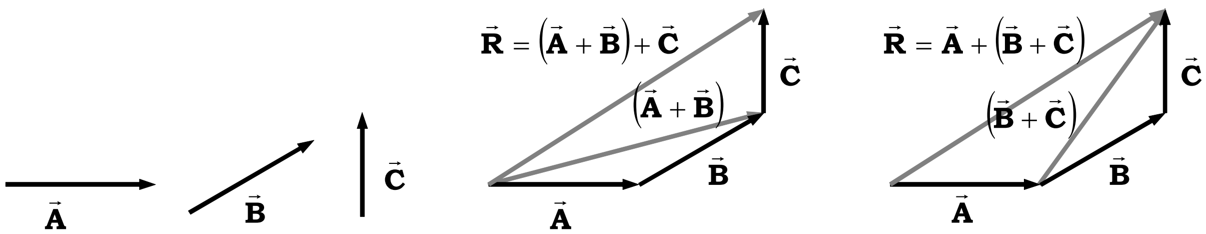
from the above diagram it is clear that

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$

hence vector addition follows law of commutation.

Vector addition is associative :

If we add more than tow vectors



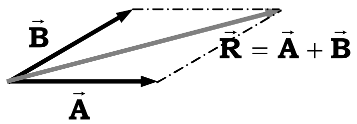
from the above diagram it is clear that

$$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) + \vec{\mathbf{C}} = \vec{\mathbf{A}} + \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right)$$

hence vector addition follows law of association.

<u>Parallelogram law of addition of two vectors</u> :

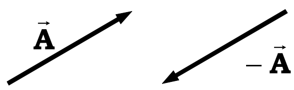
If two adjacent sides of a parallelogram represent vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ then the concurrent diagonal of the parallelogram represents the resultant of the two vectors.



As far as addition of vectors is concerned, a vector can be moved from one place to the other without changing its magnitude and direction, the result of addition of vectors does not change.

<u>Negation</u> :

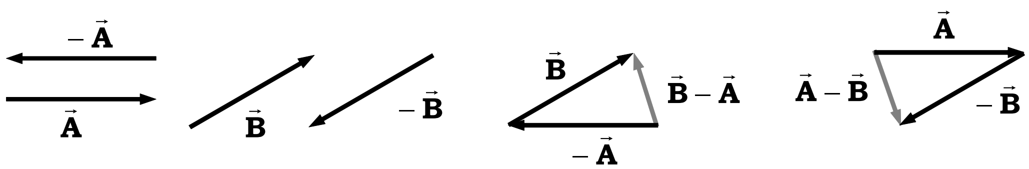
When a vector $\vec{\mathbf{A}}$ is negated it is written as $-\vec{\mathbf{A}}$. It is a vector with same magnitude but exactly opposite direction.



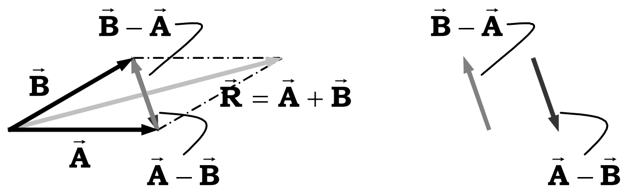
<u>Subtraction</u> :

The vector subtraction is actually the negation followed by addition.

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$$



If two adjacent sides of a parallelogram represent vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ then the non-concurrent diagonals of the parallelogram represent the subtraction $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)$ or $\left(\vec{\mathbf{B}}-\vec{\mathbf{A}}\right)$ depending on the direction of the resultant vector along the non-concurrent diagonal.



from the above discussion it is clear that

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \left|\vec{\mathbf{B}}-\vec{\mathbf{A}}\right| \text{ but}$$

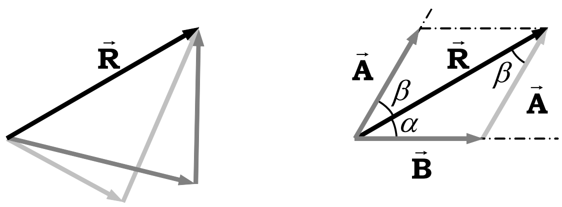
$$\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right) = -\left(\vec{\mathbf{B}}-\vec{\mathbf{A}}\right)$$

$$\Rightarrow \quad \left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right) \neq \left(\vec{\mathbf{B}}-\vec{\mathbf{A}}\right)$$

therefore vector subtraction is non-commutative.

<u>Dissociation</u> :

It is a process opposite to the addition. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are to be found whose resultant is a vector $\vec{\mathbf{R}}$. There can be infinite pair of vectors whose resultant will be $\vec{\mathbf{R}}$. Now $\vec{\mathbf{R}}$ is to be dissociated in to two vectors in predefined directions at the angle α and β from $\vec{\mathbf{R}}$.



As shown in the diagram both the resolved parts of $\vec{\mathbf{R}}$ cannot be on the same side of the $\vec{\mathbf{R}}$. Applying sin rule in the triangle of the above diagram we get the relation between the magnitudes of A, B and R

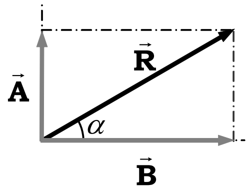
$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{R}{\sin(\alpha+\beta)}$$

$$\Rightarrow \quad A = \frac{R\sin\alpha}{\sin(\alpha+\beta)} \text{ and}$$

$$B = \frac{R\sin\beta}{\sin(\alpha+\beta)}$$

<u>Normal dissociation</u> :

When a vector $\vec{\mathbf{R}}$ is to be dissociated in two parts such that they are mutually perpendicular, the resolved parts are known as the components of vector $\vec{\mathbf{R}}$. From the adjoining diagram



$$\frac{A}{R} = \sin\alpha$$

$\Rightarrow \quad A = R\sin\alpha$

$$\frac{B}{R} = \cos\alpha$$

$\Rightarrow \quad B = R\cos\alpha$

Component of a vector by the side of the angle is the cosin component and the component opposite to the angle is the sin component.

From the above expressions of dissociation of vectors putting $\alpha + \beta = \pi/2$ we get

$$A = \frac{R\sin\alpha}{\sin(\alpha+\beta)}$$

$\Rightarrow \quad A = R\sin\alpha$

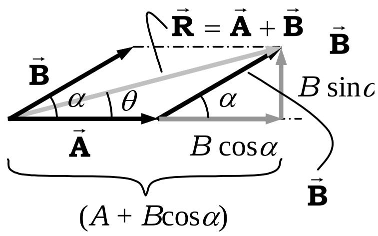
and $\quad B = \dfrac{R\sin\beta}{\sin(\alpha+\beta)}$

$\Rightarrow \quad B = R\sin\left(\pi/2 - \alpha\right)$

$\Rightarrow \quad B = R\cos\alpha$

<u>Magnitude and direction of resultant of two vectors</u> :

Let us resolve $\vec{\mathbf{B}}$ in to two components, one along $\vec{\mathbf{A}}$ ($B\cos\alpha$) and the other perpendicular to $\vec{\mathbf{A}}$ ($B\sin\alpha$). Then applying Pythagoras theorem



$$R^2 = (A + B\cos\alpha)^2 + (B\sin\alpha)^2$$

$\Rightarrow \quad R = \sqrt{A^2 + B^2\cos^2\alpha + 2AB\cos\alpha + B^2\sin^2\alpha}$

$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\alpha}$

and the angle θ between resultant $\vec{\mathbf{R}}$ and $\vec{\mathbf{A}}$

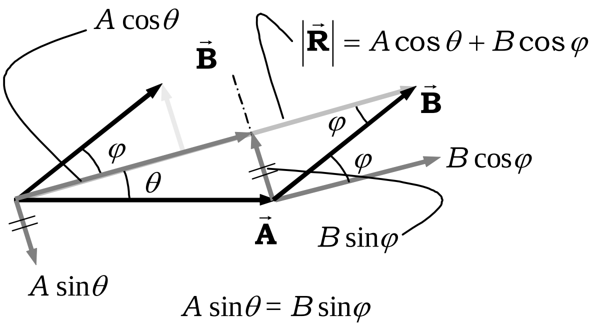
$$\tan\theta = \frac{B\sin\alpha}{A + B\cos\alpha}$$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{B\sin\alpha}{A + B\cos\alpha}\right)$

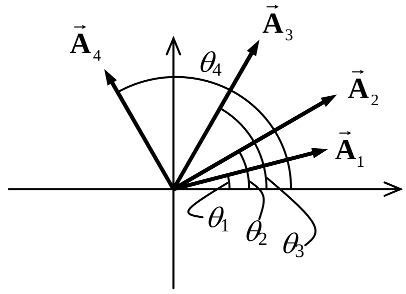
Similarly the angle θ' between resultant $\vec{\mathbf{R}}$ and $\vec{\mathbf{B}}$

$$\theta' = \tan^{-1}\left(\frac{A\sin\alpha}{B + A\cos\alpha}\right)$$

Resultant of two vectors is the resultant of their components :



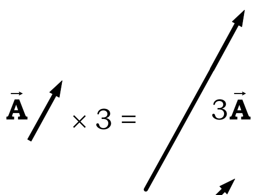
Adding more than two vecotors :
Break all the vectors into their X and Y components (if the vectors are non coplanar then Z component is also needed) and then find resultant of these components.

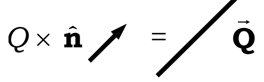


$$X = A_1\cos\theta_1 + A_2\cos\theta_2 + A_3\cos\theta_3 + A_4\cos\theta_4$$
$$Y = A_1\sin\theta_1 + A_2\sin\theta_2 + A_3\sin\theta_3 + A_4\sin\theta_4$$
and $R = \sqrt{X^2 + Y^2}$,
$$\theta = \tan^{-1}\left(Y/X\right)$$

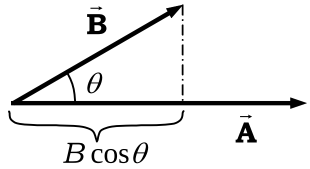
For the magnitude of resultant R to be zero, the magnitudes of X and Y both components should be separately zero.

Multiplication of a vector with a scalar :
When a vector is multiplied with a scalar the resultant is a vector with magnitude, scalar times the magnitude of original vector and direction same as the direction of original vector.
The unit vector is used to convert a scalar in to a vector in its own direction. When a unit vector $\hat{\mathbf{n}}$ is multiplied to a scalar Q, we get $\vec{\mathbf{Q}}$ whose magnitude is Q and direction is the direction of $\hat{\mathbf{n}}$.





Multiplication of a vector with a vector having result as scalar (Dot product) :
Dot product of two vectors is defined as a scalar that is the product of magnitude of one of the vector and the magnitude of component of other vector in the direction of the first vector.



$\vec{\mathbf{A}}.\vec{\mathbf{B}} = AB\cos\theta$

$\Rightarrow \quad \vec{\mathbf{A}}.\vec{\mathbf{B}} = A(B\cos\theta)$

Dot product follows law of commutation :

$\vec{\mathbf{A}}.\vec{\mathbf{B}} = A(B\cos\theta) = B(A\cos\theta) = \vec{\mathbf{B}}.\vec{\mathbf{A}}$

$\Rightarrow \quad \vec{\mathbf{A}}.\vec{\mathbf{B}} = \vec{\mathbf{B}}.\vec{\mathbf{A}}$

Dot product is distributive :
Dot product is distributive over addition.

$\vec{\mathbf{A}}.\left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \vec{\mathbf{A}}.\vec{\mathbf{B}} + \vec{\mathbf{A}}.\vec{\mathbf{C}}$

If the dot product of two vectors with non zero magnitude is zero then the vectors are mutually perpendicular.

$\vec{\mathbf{A}}.\vec{\mathbf{B}} = 0$

$\Rightarrow \quad AB\cos\theta = 0$

since A and B both are non zero therefore

$\cos\theta = 0 = \cos\left(\pi/2\right)$

$\Rightarrow \quad \theta = \pi/2$

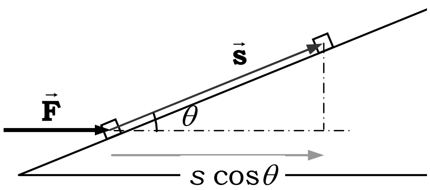
Dot product of a vector with itself is square of its magnitude and the reverse is also true.

$\vec{\mathbf{A}}.\vec{\mathbf{A}} = A^2$

Dot product is used to calculate angle between the two vectors.

$$\cos\theta = \frac{\vec{\mathbf{A}}.\vec{\mathbf{B}}}{AB}$$

Dot product in physics :
Work done by a force is defined as the dot product of force vector and the displacement vector of point of application of force.

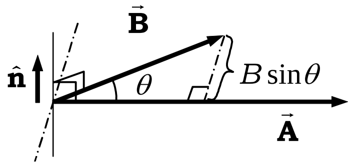


$w = \vec{\mathbf{F}}.\vec{\mathbf{s}}$

$\Rightarrow \quad w = Fs\cos\theta$

work done by a force is a scalar quantity. It is the product of magnitude of force and the magnitude of component of displacement in the direction of force.

<u>Multiplication of a vector with a vector having result as vector</u> (Cross product) :
Cross product of two vectors is defined as a vector whose magnitude is the product of magnitude of one vector and the magnitude of component of other vector perpendicular to the first vector, and the direction of resultant of cross product is perpendicular to the plane containing both the vectors given by right hand rule.



$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\Rightarrow \quad \vec{A}\times\vec{B} = A(B\sin\theta)\hat{n}$

Cross product does not follows law of commutation :

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

where as

$\vec{B}\times\vec{A} = BA\sin\theta\,\hat{n}' = AB\sin\theta(-\hat{n}) = -(\vec{A}\times\vec{B})$

$\Rightarrow \quad \vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

$\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ are same in magnitude but opposite in direction hence the cross product is non-commutative.

Cross product is distributive :

Cross product is also distributive over addition.

$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

If the cross product of two vectors with non zero magnitude is zero then the vectors are parallel or antiparallel.

$\vec{A}\times\vec{B} = 0$

$\Rightarrow \quad AB\sin\theta\,\hat{n} = 0$

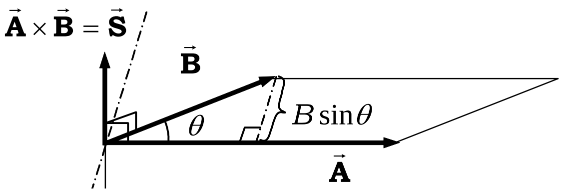
since A and B both are non zero therefore

$\Rightarrow \quad \sin\theta = 0$

$\Rightarrow \quad \theta = 0$ or $\theta = \pi$

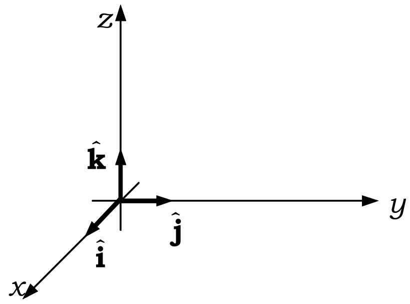
Cross product in physics :

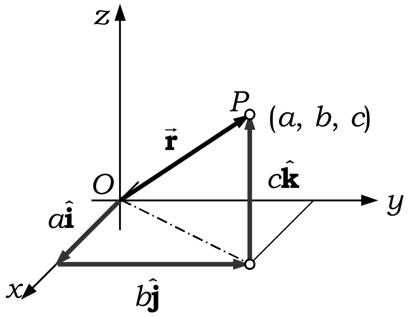
Cross product gives area vector of the parallelogram whose adjacent sides are $\vec{A}$ and $\vec{B}$. Magnitude of
cross product is $AB\sin\theta$, which is the product of base and height of the parallelogram and $\hat{n}$ is perpendicular to the surface of parallelogram giving direction to the area vector.

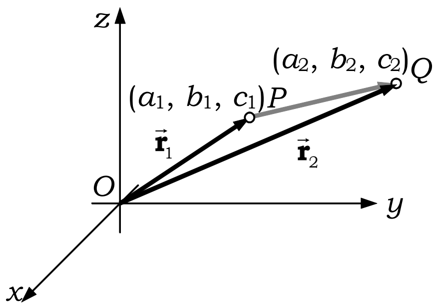


Cartesian system of vectors :

In Cartesian system of vectors we always consider a vector as the resultant of its three mutually perpendicular components along three reference axes. The unit vectors along three reference axes are taken as $\hat{i}$, $\hat{j}$ and $\hat{k}$.







From figure it is clear that

$$\vec{\mathbf{OP}} = \vec{\mathbf{r}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$$

The vector $\vec{\mathbf{OP}}$ is called position vector of point P w.r.t O.
The magnitude of $\vec{\mathbf{r}}$ is

$$r = \sqrt{a^2 + b^2 + c^2}$$

From figure

$$\vec{\mathbf{OP}} + \vec{\mathbf{PQ}} = \vec{\mathbf{OQ}}$$

$$\Rightarrow \quad \vec{\mathbf{PQ}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$$

$$\Rightarrow \quad \vec{\mathbf{PQ}} = \left(a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + c_2\hat{\mathbf{k}}\right) - \left(a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}}\right)$$

$$\Rightarrow \quad \vec{\mathbf{PQ}} = (a_2 - a_1)\hat{\mathbf{i}} + (b_2 - b_1)\hat{\mathbf{j}} + (c_2 - c_1)\hat{\mathbf{k}}$$

Operations on vectors in Cartesian system :

Let

$$\vec{\mathbf{A}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}} \quad \text{and}$$

$$\vec{\mathbf{B}} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}} \quad \text{then}$$

a) Addition :

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (a_1 + b_1)\hat{\mathbf{i}} + (a_2 + b_2)\hat{\mathbf{j}} + (a_3 + b_3)\hat{\mathbf{k}}$$

b) Negation :

$$-\vec{\mathbf{A}} = -a_1\hat{\mathbf{i}} - a_2\hat{\mathbf{j}} - a_3\hat{\mathbf{k}}$$

c) Subtraction :

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = (a_1 - b_1)\hat{\mathbf{i}} + (a_2 - b_2)\hat{\mathbf{j}} + (a_3 - b_3)\hat{\mathbf{k}}$$

d) Multiplication of a vector with scalar :

$$c\vec{\mathbf{A}} = ca_1\hat{\mathbf{i}} + ca_2\hat{\mathbf{j}} + ca_3\hat{\mathbf{k}}$$

e) Dot product :

$$\vec{\mathbf{A}}.\vec{\mathbf{B}} = \left(a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}\right).\left(b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}\right)$$

$$\Rightarrow \quad \vec{\mathbf{A}}.\vec{\mathbf{B}} = a_1b_1\hat{\mathbf{i}}.\hat{\mathbf{i}} + a_1b_2\hat{\mathbf{i}}.\hat{\mathbf{j}} + a_1b_3\hat{\mathbf{i}}.\hat{\mathbf{k}} + a_2b_1\hat{\mathbf{j}}.\hat{\mathbf{i}} + a_2b_2\hat{\mathbf{j}}.\hat{\mathbf{j}} + a_2b_3\hat{\mathbf{j}}.\hat{\mathbf{k}} + a_3b_1\hat{\mathbf{k}}.\hat{\mathbf{i}} + a_3b_2\hat{\mathbf{k}}.\hat{\mathbf{j}} + a_3b_3\hat{\mathbf{k}}.\hat{\mathbf{k}}$$

since $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are mutually perpendicular vectors therefore.

$$\hat{\mathbf{i}}.\hat{\mathbf{j}} = \hat{\mathbf{j}}.\hat{\mathbf{i}} = \hat{\mathbf{j}}.\hat{\mathbf{k}} = \hat{\mathbf{j}}.\hat{\mathbf{k}} = \hat{\mathbf{k}}.\hat{\mathbf{i}} = \hat{\mathbf{i}}.\hat{\mathbf{k}} = 0$$

and since a vector is parallel to it self

$$\hat{\mathbf{i}}.\hat{\mathbf{i}} = \hat{\mathbf{j}}.\hat{\mathbf{j}} = \hat{\mathbf{k}}.\hat{\mathbf{k}} = 1$$

hence

$$\vec{\mathbf{A}}.\vec{\mathbf{B}} = a_1b_1 + a_2b_2 + a_3b_3$$

Dot product is also used to find angle between two vectors

$$\theta = \cos^{-1}\frac{\vec{\mathbf{A}}.\vec{\mathbf{B}}}{\left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|}$$

e) Cross product :

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \left(a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}\right)\times\left(b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = a_1b_1\hat{\mathbf{i}}\times\hat{\mathbf{i}} + a_1b_2\hat{\mathbf{i}}\times\hat{\mathbf{j}} + a_1b_3\hat{\mathbf{i}}\times\hat{\mathbf{k}}$$

$$\Rightarrow \quad +a_2b_1\hat{\mathbf{j}}\times\hat{\mathbf{i}} + a_2b_2\hat{\mathbf{j}}\times\hat{\mathbf{j}} + a_2b_3\hat{\mathbf{j}}\times\hat{\mathbf{k}}$$

$$+a_3b_1\hat{\mathbf{k}}\times\hat{\mathbf{i}} + a_3b_2\hat{\mathbf{k}}\times\hat{\mathbf{j}} + a_3b_3\hat{\mathbf{k}}\times\hat{\mathbf{k}}$$

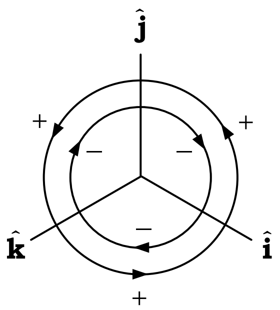
since $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are mutually perpendicular vectors therefore.

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}, \hat{\mathbf{j}}\times\hat{\mathbf{i}} = -\hat{\mathbf{k}}$$

$$\hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}, \hat{\mathbf{k}}\times\hat{\mathbf{j}} = -\hat{\mathbf{i}}$$

$$\hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}, \hat{\mathbf{i}}\times\hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

to remember this



and since a vector is parallel to it self

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = \vec{\mathbf{0}}$$

hence

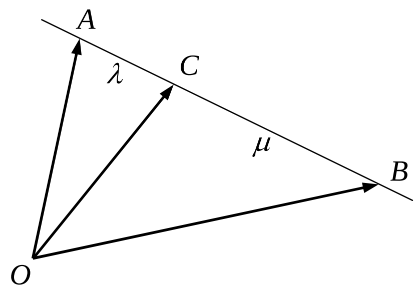
$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \vec{\mathbf{0}} + a_1b_2\hat{\mathbf{k}} + a_1b_3\left(-\hat{\mathbf{j}}\right) + a_2b_1\left(-\hat{\mathbf{k}}\right) + \vec{\mathbf{0}} + a_2b_3\hat{\mathbf{i}} + a_3b_1\hat{\mathbf{j}} + a_3b_2\left(-\hat{\mathbf{i}}\right) + \vec{\mathbf{0}}$$

$$\Rightarrow \quad \vec{\mathbf{A}}\times\vec{\mathbf{B}} = \left(a_2b_3 - a_3b_2\right)\hat{\mathbf{i}} + \left(a_3b_1 - a_1b_3\right)\hat{\mathbf{j}} + \left(a_1b_2 - a_2b_1\right)\hat{\mathbf{k}}$$

Another way to carry out the cross product is to use determinants

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \left(a_2b_3 - a_3b_2\right)\hat{\mathbf{i}} + \left(a_3b_1 - a_1b_3\right)\hat{\mathbf{j}} + \left(a_1b_2 - a_2b_1\right)\hat{\mathbf{k}}$$

λ - μ theorem :



$$\lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu)\vec{OC}$$

VECTORS

1. The magnitude of resultant of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ of equal magnitude is also same as that of the magnitude of individual vectors. The angle between the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is

A) $\pi/4$ B) $\pi/3$
C) $\pi/2$ D) $(2/3)\pi$

2. If the magnitude of vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ are 12, 5 and 13 respectively and $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ then the angle between the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is

A) $\pi/4$ B) $\pi/3$
C) $\pi/2$ D) 0

3. Two nonzero vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are such that $|\vec{\mathbf{A}} + \vec{\mathbf{B}}| = |\vec{\mathbf{A}} - \vec{\mathbf{B}}|$. The angle between them is

A) $\pi/4$ B) $\pi/3$
C) $\pi/2$ D) π

4. If the resultant of three vectors is zero then

A) magnitude of one vector must be equal to the sum of magnitudes of the other two
B) magnitude of each vector must be greater than the sum of magnitudes of the other two vectors
C) magnitude of each vector must be less than the sum of magnitudes of the other two vectors
D) all the three vectors must be of equal magnitude

5. If the dot product of the two vectors of non zero magnitude is zero than

A) they must be parallel
B) they must be anti parallel
C) they must be perpendicular
D) the angle between them depends on their magnitudes

6. The angle between two vectors $\vec{\mathbf{A}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = -8\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ is

A) $\pi/4$ B) $\pi/3$
C) π D) $\pi/2$

7. For two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$ and $A + B = C$. Angle between two vectors is

A) 0 B) $\pi/3$
C) $\pi/2$ D) π

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8. Magnitude of one vector is double that of the other and the resultant of the two vectors is equal in magnitude with the magnitude of the bigger vector. The angle between the two vectors is

A) $\cos^{-1}\left(1/2\right)$
B) $\cos^{-1}\left(-1/2\right)$
C) $\cos^{-1}\left(1/4\right)$
D) $\cos^{-1}\left(-1/4\right)$

9. The magnitude of resultant of two vectors of magnitude 5 and 10 units can not be equal to

A) 4 units
B) 6 units
C) 9 units
D) 13 units

10. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lie in a plane. Another vector $\vec{\mathbf{C}}$ lies outside this plane, then the resultant of the three vectors $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)$

A) can be of zero magnitude
B) can't be of zero magnitude
C) lies in the plane containing $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right)$
D) lies in the plane containing $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)$

11. If the dot product of the two vectors is $2\sqrt{3}$ unit and the magnitude of cross product is 2 units. The angle between the two vectors is

A) $\pi/6$
B) $\pi/4$
C) $\pi/3$
D) $\pi/2$

12. The dot product of vectors $\vec{\mathbf{A}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ is

A) 19 units
B) 3 units
C) 16 units
D) -3 units

13. Two vectors are necessarily mutually perpendicular when

A) $\vec{\mathbf{A}}+\vec{\mathbf{B}}=\vec{\mathbf{0}}$
B) $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\vec{\mathbf{0}}$
C) $\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\vec{\mathbf{0}}$
D) $\vec{\mathbf{A}}.\vec{\mathbf{B}}=0$

14. A vector $\vec{\mathbf{A}}$ is along +ve x-axis. Another vector $\vec{\mathbf{B}}$ such that $\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\vec{\mathbf{0}}$, could be

A) $4\hat{\mathbf{j}}$
B) $-4\hat{\mathbf{i}}$
C) $-\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$
D) $\left(\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$

15. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ such that $\left|\vec{\mathbf{A}}\right| = \left|\vec{\mathbf{B}}\right| = \left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right|$. The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is

A) 0
B) $\pi/3$
C) $\pi/2$
D) $\left(2/3\right)\pi$

16. If $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \vec{\mathbf{A}}.\vec{\mathbf{B}}$ than $\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right|$ is

A) $\left|\vec{\mathbf{A}}\right|+\left|\vec{\mathbf{B}}\right|$
B) $\sqrt{\left|\vec{\mathbf{A}}\right|^2+\left|\vec{\mathbf{B}}\right|^2}$
C) $\sqrt{\left|\vec{\mathbf{A}}\right|^2+\left|\vec{\mathbf{B}}\right|^2+\frac{\left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|}{\sqrt{2}}}$
D) $\sqrt{\left|\vec{\mathbf{A}}\right|^2+\left|\vec{\mathbf{B}}\right|^2+\sqrt{2}\left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|}$

17. Forces with magnitudes proportional to *AB*, *BC* and 2*CA* act along the side of a triangle *ABC* in order. Their resultant is represented in magnitude and direction by

A) *CA* B) *AC*
C) *BC* D) *CB*

18. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ of magnitude 2 units and 1 unit respectively are directed along the *x*-axis and *y*-axis. Their resultant $\vec{\mathbf{A}}+\vec{\mathbf{B}}$ is directed along the line

A) $y-2x=0$ B) $2y-x=0$
C) $y+x=0$ D) $y-x=0$

19. If $\vec{\mathbf{A}}=5\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}}=2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+a\hat{\mathbf{k}}$ are perpendicular vectors then value of a is

A) –2 B) 8
C) –7 D) –8

20. Two vectors have their magnitude in the ratio 3 : 5 and angle between their direction is $\pi/3$. If their resultant is 35 units than their magnitudes are

A) 12, 20 units B) 15, 25 units
C) 18, 30 units D) 21, 28 units

21. Five coplanar forces of equal magnitudes 10 N each, act at a point such that angle between any two consecutive forces is same. The magnitude of their resultant

A) 0 B) 10 N
C) 20 N D) $10\sqrt{2}$ N

22. A body, constrained to move in the y-direction, is subjected to a force $\vec{\mathbf{F}}=-6\hat{\mathbf{i}}+8\hat{\mathbf{j}}$ N. The work done by this force in moving the body a distance of 10 m along *y*-axis is

A) 100 J B) 140 J
C) 80 J D) 20 J

23. Three vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ satisfy $\vec{\mathbf{A}}.\vec{\mathbf{B}}=0$ and $\vec{\mathbf{A}}.\vec{\mathbf{C}}=0$, the vector $\vec{\mathbf{A}}$ is parallel to

A) $\vec{\mathbf{B}}$ B) $\vec{\mathbf{C}}$
C) $\vec{\mathbf{B}}.\vec{\mathbf{C}}$ D) $\vec{\mathbf{B}}\times\vec{\mathbf{C}}$

24. Which of the following vectors is perpendicular to the vector $4\hat{\mathbf{i}}-3\hat{\mathbf{j}}$

A) $4\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ B) $6\hat{\mathbf{i}}$
C) $7\hat{\mathbf{k}}-3\hat{\mathbf{i}}$ D) $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$

25. A particle moves from position $\vec{\mathbf{r}}_1=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-6\hat{\mathbf{k}}$ to position $\vec{\mathbf{r}}_2=14\hat{\mathbf{i}}+13\hat{\mathbf{j}}+9\hat{\mathbf{k}}$ under the action of a force $\vec{\mathbf{F}}=4\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}$. The work done by force is

A) 50 units B) 75 units
C) 100 units D) 200 units

26. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ such that $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right)\perp\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)$. Then

A) $\vec{\mathbf{A}} \,||\, \vec{\mathbf{B}}$ B) $\vec{\mathbf{A}}\perp\vec{\mathbf{B}}$
C) $\left|\vec{\mathbf{A}}\right|=\left|\vec{\mathbf{B}}\right|$ D) $\left|\vec{\mathbf{A}}\right|\neq\left|\vec{\mathbf{B}}\right|$

27. If $\vec{\mathbf{A}} = 5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 15\hat{\mathbf{i}} + 21\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ are parallel vectors then the value of a is

A) –3 B) 9
C) –9 D) 3

28. The shortest distance a spider should crawl from one corner to opposite corner of a cubical room of dimensions 3 m is

A) $3\sqrt{2}$ m B) $3\sqrt{3}$ m
C) $3\sqrt{5}$ m D) 9 m

29. The magnitude of component of $\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ in the direction of $\vec{\mathbf{B}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ is

A) 3 B) $\frac{7}{5}$
C) $\frac{7}{\sqrt{2}}$ D) 7

30. The unit vector in the direction of $\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ is

A) $\hat{\mathbf{A}} = \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{7}$ B) $\hat{\mathbf{A}} = \frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{5}$
C) $\hat{\mathbf{A}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ D) $\hat{\mathbf{A}} = \frac{4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}}{5}$

31. The maximum value of magnitude of $(\vec{\mathbf{A}} - \vec{\mathbf{B}})$ is

A) $A - B$ B) A
C) $A + B$ D) $\sqrt{A^2 + B^2}$

32. The minimum number of coplanar vectors having different non zero magnitudes can be added to give zero resultant is

A) 2 B) 3
C) 4 D) 5

33. The expression $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ is a

A) unit vector B) null vector
C) vector of magnitude $\sqrt{2}$ D) vector of magnitude $\frac{1}{\sqrt{2}}$

34. The angle between $\vec{\mathbf{R}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and y-axis is

A) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$ B) $\cos^{-1}\left(\frac{3}{\sqrt{29}}\right)$
C) $\sin^{-1}\left(\frac{3}{\sqrt{29}}\right)$ D) $\tan^{-1}\left(\frac{3}{\sqrt{29}}\right)$

35. The vector $\vec{\mathbf{A}} = 0.4\hat{\mathbf{i}} + 0.8\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ represents a unit vector, than c must be

A) –0.2 B) $\sqrt{0.2}$
C) $\sqrt{0.8}$ D) 0

36. In a clock-wise system

A) $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$ B) $\hat{\mathbf{i}}.\hat{\mathbf{i}} = 0$

C) $\hat{\mathbf{j}} \times \hat{\mathbf{j}} = 1$ D) $\hat{\mathbf{k}}.\hat{\mathbf{i}} = 1$

37. A bus is going due North at a speed of 50 km h^{-1}. It makes a 90^o left turn without changing the speed. The change in the velocity of the bus is

A) 50 km h^{-1} toward West B) 50 km h^{-1} towards South-West

C) 70.7 km h^{-1} towards South-West D) 70.7 km h^{-1} towards North-West

38. The angle between two vectors $-2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\bar{\mathbf{j}} - 4\hat{\mathbf{k}}$ is

A) 0^o B) 90^o

C) 180^o D) none

39. A body moving in circular motion with constant speed has

A) constant velocity B) constant acceleration

C) constant kinetic energy D) constant displacement

40. While traveling from one station to another, a car travels 75 km North 60 km North-East and 20 km East. The distance between the two stations is

A) 72 km B) 112 km

C) 133 km D) 155 km

41. What happens, when we multiply a vector by (–2)

A) direction reverses and unit changes

B) direction reverses and magnitude is doubled

C) direction remains unchanged but unit changes

D) neither direction reverses not unit changes but magnitude is doubled

42. A particle moves towards east with a speed 5 m/s. After 10 seconds its direction changes towards north with speed being same. The average acceleration of the particle is

A) zero B) $\frac{1}{\sqrt{2}}$ m s^{-2} N-W

C) $\frac{1}{\sqrt{2}}$ m s^{-2} N-E D) $\frac{1}{\sqrt{2}}$ m s^{-2} S-W

43. A particle has a displacement of 12 m towards east and 5 m towards north then 6 m vertically upwards. The magnitude of resultant displacement is

A) 14.32 m B) 12 m

C) 10.04 m D) none of these

44. The resultant of two vectors acting at an angle of 150^o is of magnitude 10 units and is perpendicular to one of the vectors. The magnitude of the other vector is

A) $\frac{20}{\sqrt{3}}$ N B) $10\sqrt{3}$ N

C) 20 N D) $20\sqrt{3}$ N

45. If a body placed at the origin is acted upon by a force $\vec{\mathbf{F}} = \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \sqrt{2}\,\hat{\mathbf{k}}\right)$, then which of the following statements are correct

A) magnitude of $\vec{F}$ is $\left(2+\sqrt{2}\right)$

B) magnitude of $\vec{F}$ is $\left(2\sqrt{2}\right)$

C) $\vec{F}$ makes an angle of 45° with the z-axis

D) $\vec{F}$ makes an angle of 30° with the z-axis

46. Magnitude of a vector which comes on addition of two vectors $6\hat{i}+7\hat{j}$ and $3\hat{i}+4\hat{j}$ is

A) $\sqrt{136}$ B) $\sqrt{132}$

C) $\sqrt{202}$ D) $\sqrt{160}$

47. Let $\vec{A}=A\cos\theta\hat{i}+A\sin\theta\hat{j}$ be any vector. Another vector $\vec{B}$, which is normal to $\vec{A}$ can be expressed as

A) $\hat{i}\,B\cos\theta-\hat{j}\,B\sin\theta$ B) $\hat{i}\,B\cos\theta+\hat{j}\,B\sin\theta$

C) $\hat{i}\,B\sin\theta-\hat{j}\,B\cos\theta$ D) $\hat{i}\,B\sin\theta+\hat{j}\,B\cos\theta$

48. The angle between $\vec{P}$ and $\vec{Q}$ is θ. $\vec{R}=\vec{P}+\vec{Q}$ makes angle $\theta/2$ with $\vec{P}$. Which of the following is true

A) $P = 2Q$ B) $2P = Q$

C) $P\,Q = 1$ D) None

49. What is the value of $(\vec{P}+\vec{Q})\times(\vec{P}-\vec{Q})$

A) 0 B) $P^2 - Q^2$

C) $P^2 + Q^2 + 2PQ$ D) None of these

50. The resultant of $\vec{A}\times\vec{0}$ will be equal to

A) zero B) A

C) zero vector D) unit vector

Answers to Vectors

01. D)	***02.*** C)	***03.*** C)	***04.*** C)	***05.*** C)	***06.*** C)	***07.*** A)	***08.*** D)	***09.*** A)	***10.*** B)
11. A)	***12.*** D)	***13.*** D)	***14.*** B)	***15.*** B)	***16.*** D)	***17.*** A)	***18.*** B)	***19.*** B)	***20.*** B)
21. A)	***22.*** C)	***23.*** D)	***24.*** D)	***25.*** C)	***26.*** C)	***27.*** C)	***28.*** C)	***29.*** C)	***30.*** B)
31. C)	***32.*** A)	***33.*** A)	***34.*** B)	***35.*** B)	***36.*** A)	***37.*** C)	***38.*** D)	***39.*** C)	***40.*** D)
41. B)	***42.*** B)	***43.*** A)	***44.*** C)	***45.*** C)	***46.*** C)	***47.*** C)	***48.*** D)	***49.*** D)	***50.*** C)