DIMENSIONAL & ERROR ANALYSIS

Physical quantity :
Any number that is used to describe a physical phenomenon quantitatively is called a physical quantity.

Measurement of a Physical Quantity :
The process of measurement is basically comparison process. To measure a physical quantity, we have to find out how many times a standard amount of that physical quantity is present in the quantity being measured. The number thus obtained is known as the magnitude and the standard chosen for comparison is called the unit of the measurement.

The standards :
when we measure a quantity, we always compare it with some reference standard, which does not change. Standards should be

i) invariable
ii) easily accessible
iii) precise
iv) universally agreed upon

The unit :
The unit of a physical quantity is a value of it in terms of which other quantities of that kind are expressed. It is chosen arbitrarily but then agreed upon universally.

System of units :
There are three popular system of units

i) **CGS system** – The units of length, mass, time are centimeter, gram, second respectively.
ii) **FPS system** – The units of length, mass, time are foot, pound, second respectively.
iii) **SI system** – The units of length, mass and time are meter, kilogram, second respectively also called standard international system of units.

Fundamental Quantities and their SI units :
A fundamental physical quantity is the quantity which does not depend on any other physical quantity. There are six fundamental quantities in SI system. They are

S.N.	Physical quantity	unit	dimension
1.	length,	meter	[L]
2.	mass,	kilogram	[M]
3.	time,	second	[T]
4.	electric current,	ampere	[A]
5.	temperature and	Kelvin	[K]
6.	amount of substance	mole	[mol]
7.	luminous intensity.	candela	[I]

In addition there are two supplementary quantities

S.N.	Physical quantity	unit	dimension
1.	angle in a plane,	radian	[θ]
2.	solid angle	ste-radian	[ω]

Definitions of SI units of fundamental quantities **:**

i) **metre :**
1 metre is defined as 1,650,763.73 wavelengths of radiation corresponding to orange red light of krypton-86 in free space.

ii) **second :**
1 second is defined as 9,192631,770 time periods of oscillations of cesium-133 atom.

iii) **kilogram :**
1 kg is defined as the mass of 1 litre of pure water at 4 ^{o}C and 1 atm pressure.

iv) **ampere :**
1 ampere is defined as the current flowing through two infinitely long, thin, parallel wires placed at a separation of 1 m in free space interacting with a force of 2×0^{-7} m^{-1}(per meter length of wire).

v) **Kelvin :**
1 K is defined as $\left(\frac{1}{273.16}\right)^{th}$ part of the thermodynamic temperature of triple point of water.

vi) **mole :**
1 mole is defined as amount of substance containing as many particles as 12 gm of carbon-12.

vii) **candela :**
1 candela is defined as the luminous intensity in a direction perpendicular to $\left(\frac{1}{600000}\right)m^2$ surface of a black body at the temperature of freezing platinum at a pressure 1.013×10^5 N m^{-2}.

viii) **radian :**
1 radian is defined as plane angle between two radii of a circle with length of arc of circle between them equal to radius of the same circle.

ix) **steradian :**
1 steradian is defined as the solid angle subtended by an area on the surface of a sphere equal to area of a square with side equal to the radius of sphere at the centre of sphere.

Prefixes for SI units :

S.N.	Symbol Prefix		Multiple
1.	E	exa	10^{18}
2.	P	peta	10^{15}
3.	T	tera	10^{12}
4.	G	giga	10^{9}
5.	M	mega	10^{6}
6.	K	kilo	10^{3}
7.	M	milli	10^{-3}
8.	μ	micro	10^{-6}
9.	N	nano	10^{-9}
10.	P	pico	10^{-12}
11.	F	femto	10^{-15}
12.	A	atto	10^{-18}

Accepted but not recommended ones :

S.N.	Symbol Prefix		Multiple
1.	H	hecto	10^{2}
2.	Da	deca	10^{1}
3.	D	deci	10^{-1}
4.	C	centi	10^{-2}

Derived SI units :
These are the units for derived quantities and consist of the base units (fundamental units) in various combinations. Many important derived units have been given names of scientists e.g. Newton, joule etc.

Some important practical units :
To measure very large distances following units are used

a) Astronomical unit(Au) : It is the mean distance of the earth from the sun.
$$1 \text{ Au} = 1.496 \times 10^{11} \text{ m} = 1.5 \times 10^{11} \text{ m}.$$
b) Light year (Ly) : It is defined as the distance traveled by light in vacuum in one year.
$$1 \text{ light year} = 9.5 \times 10^{15} \text{ m}$$
c) Parsec : A parsec is the distance at which an arc of length one astronomical unit subtends an angle of one second.
$$1 \text{ Parsec} = 3.084 \times 10^{16} \text{ m} = 3.1 \times 10^{16} \text{ m}$$

Relation between Au, Ly and Parsec :

1 Ly = 6.3 104 Au, 1 Parsec = 3.26 Ly,

To measure very small distances :

a) 1 micron (1μ or $1\mu m$) = 10^{-6} m
b) 1 angstrom (1 A°) = 10^{-10} m
c) 1 fermi or femometer (1 fm) = 10^{-15} m
d) 1 barn (1 b) = 10^{-28} m^2

To measure very large masses :

a) 1 Slug = 14.57 kg
b) 1 Quintal = 100kg
c) 1 Ton (or metric ton) = 10 quintal = 1000kg

To measure very small masses :

a) 1 a.m.u. or 1u = 1.66×10^{-27}kg

Dimensions, Dimensional Formulae and Dimensional Equations :

Dimension- of a physical quantity is the power to which the basic unit should be raised to express its derived unit.

Dimensional Formula- is the expression representing the dimension of a physical quantity.

Dimensional Equation- is the equation obtained when a physical quantity is equated with its dimensional formula.

Uses of Dimensional Equations:

a) Conversion of one system of units to another

If dimensional formula of a physical quantity is $[M^a L^b T^c]$ and its derived units in the two systems are

$[M_1^a L_1^b T_1^c]$ and $[M_2^a L_2^b T_2^c]$, then

$n_1[M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

where n_1 & n_2 are numerical values in the two systems. e.g. To convert 1N = kg ms^{-1} to g cms^{-2}

$n_1[\text{kg}] [\text{m}] [\text{s}]^{-2} = n_2 [\text{g}] [\text{cm}] [\text{s}]^{-2}$

or $n_2 = n_1 \dfrac{[\text{kg}]}{[\text{g}]} \times \dfrac{[\text{m}]}{[\text{cm}]} \times \dfrac{[\text{s}]^{-2}}{[\text{s}]^{-2}} = (10^3 \times 10^2 \times 1)\, n_1$

or 1N = 10^5 dyne

b) To check the correctness of a physical relation

This is based on the principle of homogeneity of dimensions which states that dimensions of all the term on the two sides of an equation must be the same. A dimensionally correct relation may actually be incorrect, e.g. strain ≠ relative density even though this relation is dimensionally correct. (both strain & relative density are dimensionless).

c) To derive new relations

If a physical quantity X depends on other physical quantities P, Q, R i.e.

$X = P^a Q^b R^c$ ... (1)

Where a, b, c are powers raised on physical quantities P, Q, R respectively. Then, these physical quantities are written in terms of the dimensional formula and dimensions are equated on either side. This determines values of a, b, c which if substituted in (1) gives new dimensional relation.

Limitations of dimensional analysis :

The limitations of dimensional analysis are

1. It gives no information about pure numeric and non-dimensional constants.
2. It gives no information about the dependence of a physical quantity on trigonometric, exponential & logarithmic functions.
3. If a physical quantity depends upon more than three physical quantities then dimensional analysis can not be used to derive their relationship.
4. This method gives no information about dimensional constants or other constants such as the universal constant of gravitation (G) and Planck's constant (h)

ERROR ANALYSIS

Significant Figures :

The numbers of digits in the measured value about the correctness of which we are sure plus one more uncertain or doubtful digit are called significant figures. Or in other words we can say, the digits whose values are accurately known in a particular measurement are called its significant figures.

Rules for determining the significant figures :

1. The number of significant figures are counted from the first non-zero digit on the left to the last digit on the right.
2. All non-zero digits are significant.
3. All zeros occurring between two non-zero digits are significant, e.g., both 230089 and 2.30089 contain 6 significant figures.
4. All zeros to the right of a decimal point and to the left of a non-zero digit are significant.
5. All the zeros to the right of the last non - zero digit (terminal trailing zeros) in a number without a decimal point are not significant. 20 m, 2000 cm, 2000 mm all have just 1 significant figure.
6. The trailing zeros in a number with a decimal point are significant. Therefore 3.700 or 0.004700 have 4 significant figures.
7. All zeros to the right to the last non-zero digit are significant, if they come from a measurement.

Do not retain a greater number of decimal places in a result computed from arithmetic operations than in the observation that has the finest decimal places.

Adding or subtracting the numbers :

The number of decimal places in the answer should agree with the number of places in the term that has least number of decimal places. Thus 472 + 3.64 is not written as 475.64 but is rounded off to 476. however, if we have the subtraction ,

$$8.5675 \text{ kg} - 8.556\text{kg} = 0.012\text{kg},$$

the answer has only two significant figures even though the input data had 5 & 4 significant figures respectively.

When we add or subtract values, we must pay careful attention to the number of decimal places rather the number of significant figures. The number of decimal places in the answer is equal to the smallest number of decimal places in any of the values added or subtracted.

Multiplication or division of numbers :

The result obtained should be stated in the minimum number of significant figures which any of the observation has. *A chain cannot be stronger than its weakest link.*

Rounding off :

i) If the digit next to the one being rounded is more than 5, the digit to be rounded is increased by 1. e.g., 9.36, 3 is the digit being rounded & 6 being > 5, 3 is increased by 1 to 4 hence 9.36 after rounding off becomes 9.4.

ii) If the digit next to the one being rounded is less than 5, the digit rounded is left unchanged, e.g., in 5.74, the digit being rounded is 7 & the next digit 4 < 5, the digit 7 is left unchanged and digit 4 is dropped. 5.74 after rounding off become 5.7.

iii) a) If the digit next to the one being rounded is 5, the digit being rounded is increased by 1 if it is odd, e.g., if the digit to be rounded is 7 in 6.875, answer will be 6.88.

b) If the digit next to the one being rounded is 5, the digit being rounded is left unchanged if it is even, e.g., in 8.765, if the digit being rounded is 6, the digit next to it is 5, and answer will be 8.76.

Types of error :

1. Constant errors :
when the results of a series of observations are in error by the same amount, the error is said to be a constant.

2. Systematic errors :
A systematic error is one that always produces an error of the same sign. It may be instrumental error, observation error, error due to external causes, or due to imperfection.

3. Random errors :
These are the errors of chance or due to unknown causes. If a_1, a_2, a_3........a_n are different readings then arithmetical mean will be

$$\bar{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

4. Gross error :
These are the results of shear carelessness on the part of the person performing experiment

5. Absolute error :
The magnitude of thedifference of the true value and the measured value is called absolute error.
Absolute error = true value – measured value

$$\Delta a_i = \bar{a} - a_i$$

6. Percentage error :

$$\left[\frac{\Delta\bar{a}}{\bar{a}}\right] \times 100 \text{, where } \bar{a} = \text{arithmetic mean}$$

7. Relative error :
It is defined as the ratio of the absolute error to the true value.

$$S_a = \frac{\Delta\bar{a}}{\bar{a}}$$

Combination of errors :

a) In sum and difference
If result

$z = a \pm b$,

where a and b are two measurements then maximum absolute error involved is

$\Delta z = \Delta a + \Delta b$

b) In multiplication or division

If $z = ab$ or $z = a/b$

Then maximum relative error in z involved would be

$\Delta z/z = (\Delta a/a) + (\Delta b/b)$

c) In power

If $z = \frac{a^p b^q}{c^r}$, the maximum relative error in z is

$\Delta z/z = p(\Delta a/a) + q(\Delta b/b) + r(\Delta c/c)$

and actual (likely) percentage error will be

$p(\Delta a/a) \times 100 + q(\Delta b/b) \times 100 + r(\Delta c/c) \times 100$

Formulae and dimensions of physical quantities :

S.N.	Physical quantity (symbol)	Relation with other Physical quantities	SI Unit	Dimensional Formula
1.	Area (A)	length × breadth	m^2	$M^0L^2T^0$
2.	Volume (V)	length × breadth × height	m^3	$M^0L^3T^0$
3.	Velocity (v)	distance / time	m/s	$M^0L^1T^{-1}$
4.	Acceleration (a)	change in velocity / time	m/s^2	$M^0L^1T^{-2}$
5.	Force (F)	mass × acceleration	kgm/s^2	$M^1L^1T^{-2}$
6.	Pressure (p, P)	force / area	N/m^2	$M^1L^{-1}T^{-2}$
7.	Momentum (p)	mass × velocity	kgm/s = Ns	$M^1L^1T^{-1}$
8.	Impulse	force × time	kgm/s = Ns	$M^1L^1T^{-1}$
9.	Work (w)	Force × distance	kgm^2/s^2	$M^1L^2T^{-2}$
10.	Kinetic energy (K_E)	½ mass×velocity²	kgm^2/s^2	$M^1L^2T^{-2}$
11.	Potential energy (P_E)	force × distance	kgm^2/s^2 = J	$M^1L^2T^{-2}$
12.	Torque (τ)	force × length of arm	Nm	$M^1L^2T^{-2}$
13.	Power (P)	work / time	kgm^2/s^2= J/s	$M^1L^2T^{-3}$
14.	Angle (θ, φ)	arc / radius	radian	$M^0L^0T^0$
15.	Solid angle (Ω)	area / (⊥dis.)²	steradian	$M^0L^0T^0$
16.	Angular velocity (ω)	angular displacement/time	rad/s	$M^0L^0T^{-1}$
17.	Angular acceleration (α)	change in angular velocity/time	rad/s^2	$M^0L^0T^{-2}$
18.	Frequency (f, ν)	number of vibrations/time	hertz(Hz)	$M^0L^0T^{-1}$

19.	Radius of gyration (*k*)	Distance	m	$M^0L^1T^0$
20.	Moment of inertia (*I*)	mass × distance²	kgm²	$M^1L^2T^0$
21.	Angular momentum (*L*)	moment of inertia × angular velocity	kgm²/s = Js	$M^1L^2T^{-1}$
22.	Surface tension (*T*)	force / length	N/m	$M^1L^0T^{-2}$
23.	Stress	force / area	N/m²	$M^1L^{-1}T^{-2}$
24.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	No units	$M^0L^0T^0$
25.	Modulus of elasticity	stress / strain	N/m²	$M^1L^{-1}T^{-2}$
26.	Density (ρ)	mass / volume	kg/m³	$M^1L^{-3}T^0$
27.	Gravitational constant (*G*)	$\frac{F_g r^2}{m_1 m_2}$	Nm²/kg²	$M^{-1}L^3T^{-2}$
28.	Surface energy	Energy/area	J/m²	$M^1L^0T^{-2}$
29.	Velocity gradient	velocity/distance	s^{-1}	$M^0L^0T^{-1}$
30.	Coefficient of Viscosity (η)	$\frac{\text{force}}{(\text{area} \times \text{velocity gradient})}$	daP	$M^1L^{-1}T^{-1}$
31.	Relative density	$\frac{\text{Density of material}}{\text{Density of water at } 4°\text{C}}$	No units	$M^0L^0T^0$

Least Count :

The minimum quantity that can be measured accurately by an instrument is called the least count. The least count of a metre scale graduated in millimeter marks is 1 mm. The least count of a watch having second's hand is 1 second.

- least count of vernier calipers

  = length of one part on main scale – length of one part on vernier scale.

  $= \frac{\text{length of one part on main sacle}}{\text{number of parts on vernier scale}}$

- least count of screw gauge

  $= \frac{\text{length of pitch}}{\text{number of parts on circular scale}}$