DIMENSIONAL & ERROR ANALYSIS

1. The Kinetic Energy E_k of a rotating body depends on its moment of inertia I and its angular speed ω. Assuming the relation to be $E_k = k\, I^a \omega^b$ where k is a dimensionless constant. Find a and b. Moment of inertia of a sphere about its diameter is $\left(\frac{2}{5}\right) MR^2$.

Answer : $a = 1, b = 2$

2. If density (ρ), acceleration due to gravity (g) and frequency (ν) are taken as base quantities, find the dimensions of force.

Answer : $\frac{\rho g^4}{\nu^6}$

3. If the velocity of light be represented by unity and 100 sec be the unit of time, what must be unit of length?

Answer : 3×10^{10} m

4. If the velocity of light c, gravitational constant G and planks constant h be chosen as fundamental units, find the value of a gram, a cm and a sec in term of new unit of mass, length and time respectively.
($c = 3 \times 10^{10}$ cm/sec; $G = 6.67 \times 10^{-8}$ dyn cm^2 gram^{-2}; $h = 6.6 \times 10^{-27}$ erg sec)

Answer : $1\,gm = 1.816 \times 10^4\, units, 1\,cm = 2.48 \times 10^{32}\, units, 1\,\text{sec} = 7.44 \times 10^{42}$

5. Assuming that the mass m of the largest stone that can be moved by a flowing river depend only on V the velocity, ρ density of water and g show that m varies with the sixth power of the velocity of flow.

6. Equation of wave motion is

$$y = A \sin \omega\left(\frac{x}{v} - kt\right);$$

where ω = angular velocity, v = linear velocity. What are the dimensions of x and k?

Answer : $[M^0L^1T^0]$, $[M^0L^0T^0]$

7. The radius and length of a thin wire are 0.46 mm and 2.17 m respectively. Find the volume of the wire up to appropriate number of S.F.?

Answer : $1.4\, m^3$

8. The side of cube is measured as (7.5 ± 0.1) cm. Find the volume of the cube?

Answer : $4.2 \times 10^2 \pm 17$ cm^3

9. The velocity v of a point at time t is given by

$$v = at + \frac{b}{t + c}.$$

What are the dimensions of a, b and c respectively?

Answer : $[M^0L^1T^{-2}]$, $[M^0L^1T^0]$, $[M^0L^0T^1]$

10. In case of diffusion the number of molecules crossing unit area per unit time is given by

$$N = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$$

Where n_1 and n_2 are number of molecules per unit volume at positions x_1 and x_2. Find the dimensions of coefficient of diffusion D.

Answer : $[M^0L^2T^{-1}]$

11. Turpentine oil is flowing through a tube of length l and radius r. the pressure difference between the ends of the tube is p and viscosity of oil is given by

$$\eta = p\left(\frac{r^2 - x^2}{4vl}\right)$$

where v is the velocity of oil at a distance x from the axis of the tube. Find the dimensions of viscosity.

Answer : $[M^1L^{-1}T^{-1}]$

12. Find the dimensions of a/b in the equation

$$p = \frac{a - t^2}{bx}$$

where p is pressure, x is distance, t is time.

Answer : $[M^1L^0T^{-2}]$

13. Show that dimension of

$$\frac{h}{m_0 c}$$

is of the length, h is Planck's constant, m_o rest mass and c is velocity of light.

14. Find dimensionally the relation between reverberation period t of a room, volume V, surface area A and velocity of sound c.

Answer : $t = k\frac{V}{Ac}$

15. Find the value of 60 J/min on a system, which has 100 gm, 100 cm and 1 min as fundamental units.

Answer : 2.16×10^6 units

16. Test by the method of dimensions, the accuracy of relation

$$T = 2\pi\sqrt{\frac{k^2 + l^2}{lg}}$$

for the time period of pendulum.

17. The period of oscillation of a pendulum is

$$T = 2\pi\sqrt{l/g},$$

l is about 10 cm and is known as to 1 mm accuracy. The period of oscillations is 0.5 sec. The time of 100 oscillations is measured with a wristwatch of 1 s resolution. What is the accuracy in the determination of g?

Answer : 5 %

18. Specific resistance ρ of thin circular wire of radius r cm, resistance R ohm and length l cm is given by

$$\rho = \frac{\pi r^2 R}{l}.$$

If $r = (0.26 \pm 0.02)$, $R = (32 \pm 1)$, $l = (78 \pm 0.01)$, Find the percentage error in ρ.

Answer : 18.50 %

19. A certain body weighs 22.42 gm and has a measured volume of 4.7 cc. The possible error in the measurement of mass and volume are 0.01 gm and 0.1 cc. What is maximum error in calculation of density?

Answer : 0.1 gm c^{-3}

20. A spherometer has 250 equal graduations marked along the periphery of its disc, and one full rotation of the disc advances on the main scale by 0.0625 cm. What is the least count of the system?

Answer : 2.5×10^{-4} cm.

21. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block of same dimensions and of low modulus of rigidity η such that lower face of A completely covers the upper face of B. The lower face of B is rigidity hold on a horizontal surface. A small force F is applied perpendicular to one of the side face of A. After the force is withdrawn, block A executes small oscillations. Find the time period.

Answer : $T = 2\pi\sqrt{\dfrac{M}{\eta l}}$

22. Richardson equation is given by

$$I = CT^2 e^{-B/kT}$$

where C and B are constants. Find the dimensions of C and B, where I is the current T is the Temperature and k is Boltzman's constant.

Answer : $[A^1K^{-2}]$, $[ML^2T^{-2}]$

23. Find the dimension of X where $X = \dfrac{\text{distance covered in } n^{\text{th}} \text{ sec}}{\text{distance covered in } n \text{ seconds}}$

Answer : $[M^0L^0T^0]$

24. The position of particle at time t is given by

$$x(t) = \frac{v_0}{\alpha}\left(1 - e^{-ut}\right) + \frac{\alpha}{T}$$

where v is a constant and $\alpha > 0$. Find the dimension of v_0 and α

Answer : $v_0 = [M^0L^2T^1]$

25. Column I give three physical quantities. Select appropriate units for these from choices given in column II

I			II	
			1	Ohm sec
a	Capacitance		2	$(\text{Couloumb})^2\ \text{Joule}^{-1}$
			3	$\text{Couloumb volt}^{-1}$
b	Inductance		4	$NA^{-1}m^{-1}$
c	Magnetic induction		5	Volt sec Amp^{-1}

Answer : a : 2, 3; b : 1, 5; c : 4

26. Two ceasium clocks if allowed to run for fore 10 years may differ by only about 0.02 sec. Calculate the of accuracy in measuring the time interval of 1 sec.

Answer : 6.34×10^{-11} sec in 1 sec.

27. E, m, L and G denote energy, mass, angular momentum and gravitational constant respectively. Find the dimensions of

$$\frac{EL^2}{m^5G^2}.$$

Answer : $[M^{-1}L^3T^{-2}]$

28. State No. of significant figures in

i) $0.005\ m^2$ ii) 2.63×10^{28} Kg iii) $0.2560\ \text{gm cm}^{-3}$
iv) 6.320 N v) 5.003 J vi) 0.0006032
vii) 200.05 viii) 12500 ix) 122
x) 0.08240

Answer : i) 1, ii) 3, iii) 4, iv) 4, v) 4, vi) 4, vii) 5, viii) 5, ix) 3, x) 4

29. The time period of oscillation of a simple pendulum in an experiment is recorded as 2.56 s, 2.62 s, 2.70 s, 2.58 s, 2.45 s respectively. Find time period, absolute error and percentage error.

Answer : 2.58 sec, 0.06 sec, 2.3 %

30. 1 calorie = 4.2 J where J has the dimension of energy. Now system of units is so chosen that the unit of mass equals α kg, the unit of length equals β m and the unit of time is γ s, then prove that

$$1 \text{ cal} = 4.2 \frac{\gamma^2}{\alpha\beta^2} \text{ in term of new units.}$$

31. The parallax of heavenly body measured from two points diametrically opposite on earth's equation is 60 sec. If the radius of earth is 6.4×10^6 m, determine the distance of the heavenly body from the center of the earth.

Answer : 2.2×10^{10} m

32. Total time taken by laser beam to return to the earth after reflection from the moon is 2.56 sec. Calculate the distance of the moon from the earth.

Answer : 3.84×10^8 m

33. Nuclear radius is given by

$$r = r_0 A^{1/3}$$

where r is the radius of the nucleus, A its mass number and r_0 is constant equal to 1.2×10^{-15} m. show that nuclear mass density is nearly constant for different nuclei. (1 a.m.u = 1.66×10^{-27} Kg).

Answer : 10^{17} kg m^{-3}

34. The number of minutes in one hour is 60 and number of seconds in one hour is 3600. Find their order of magnitude

Answer : 10^2 minutes, 10^4 sec.

35. The Potential energy of a particle varies with distance x from a fixed origin as

$$U = \frac{A\sqrt{x}}{x^2 + B}$$

where A and B are constants. Find the dimensional formula of AB.

Answer : $[M^1L^{5.5}T^{-2}]$

36. Planck time depends on c, G and h, where letters have their usual meaning. Find the order of magnitude of Planck time.

Answer : 10^{-43} sec.

37. In two system of units relation among velocity, acceleration and forces are

$$v_2 = \frac{\alpha^2}{\beta} v_1,\ a_2 = \alpha\beta\, a_1,\ F_2 = \frac{F_1}{\alpha\beta}.$$

If α and β are constants, then find the relation among mass, length, time in two systems.

Answer : $M_2 = \frac{M_1}{\alpha^2\beta^2},\ t_2 = \frac{\alpha t_1}{\beta^2},\ L_2 = \frac{\alpha^3 L_1}{\beta^3}$

38. A number X, which represents the age of the universe, depends upon G, c, m_e, m_p, $e, \in_0$ (where terms have their usual meaning). Given $X \propto e^4 m_e^{-2} m_p^{-1}$ Find the relation of X to other quantities.

Answer : $k\frac{e^4}{m_e^2 m_p \in_o^2 Gc^3}$

39. A quantity X is given by

$$\in_0 L\frac{\Delta V}{\Delta t}$$

where L is the length, ΔV is the potential difference and Δt is the time taken. Then find the dimensions of X and state a physical quantity, which has same dimensions as X.

Answer : [A], current

40. Give the dimensional formula of the following

a) Mechanical energy	b) Capacitance	c) Permitivity of free space
d) Magnetic induction	e) Angle of Dip	f) Coefficient of Viscosity
g) Angular momentum	h) Centripetal force	i) Torque
j) Specific heat	k) Young's Modulus	l) Plank's constant