

**MOTION IN TWO & THREE DIMENSION**

- A ball projected at an angle  $37^\circ$  attains horizontal range 120 m. What maximum height the ball achieves in this journey? (Given  $\sin 37^\circ = 0.6$ ).  
**Answer :** 22.5 m
- A particle is projected at an angle of  $45^\circ$  with a velocity  $20 \text{ ms}^{-1}$ . Find the horizontal separation between two points of same height of 5 m on its trajectory. ( $g = 10 \text{ ms}^{-2}$ ).  
**Answer :** 28.3 m
- A particle projected at an angle of projection  $60^\circ$  attains maximum height of 40 m above ground. At what distance from point of projection it will land on horizontal ground?  
**Answer :** 92.4 m
- A particle dropped from the top of a tower of height 20 m. Due to horizontal wind the particle gained a constant horizontal velocity  $10\sqrt{2} \text{ ms}^{-1}$ . There is no air resistance in vertical motion. Find
  - the horizontal distance from the foot of the tower where it lands
  - the angle from horizontal at which it strikes the horizontal ground.**Answer :** a) 28.3 m; b)  $\theta = \tan^{-1}(\sqrt{2})$
- A boy throws a ball with a velocity  $10 \text{ ms}^{-1}$  at an angle  $53^\circ$  above horizontal, and starts running in its line with uniform acceleration immediately after projecting the ball to catch it. If he could catch the ball at same height from which it was projected, with what acceleration the boy ran?  
**Answer :**  $7.5 \text{ m s}^{-2}$
- Two particles are projected from an elevated point in same vertical plane. One horizontally with a velocity  $12 \text{ ms}^{-1}$  and other with a velocity  $20 \text{ ms}^{-1}$  at  $53^\circ$  above horizontal. Find distance between them when their speeds are equal. ( $g = 10 \text{ ms}^{-2}$ )  
**Answer :** 12.8 m
- A particle is projected horizontally with a velocity  $12 \text{ ms}^{-1}$  from an elevated point. When it is moving  $53^\circ$  below horizontal (its velocity vector is  $53^\circ$  below horizontal) find the horizontal distance it has traveled till this instant. (Given  $\cos 53^\circ = 0.6$ ).  
**Answer :** 19.2 m
- A gun capable of firing shells with muzzle velocity  $500 \text{ ms}^{-1}$  fires the instant a fighter plane passes over it. At what minimum height the fighter plane flying at  $300 \text{ ms}^{-1}$  should fly to remain out of danger of being hit by shells fired from the gun.  
**Answer :** 8 km
- A boy throws a stone with a velocity  $30 \text{ ms}^{-1}$  aiming directly a bird sitting on a wall of height 20 m from ground perpendicular to plane of projectile. If the stone is released from the point 2 m above horizontal and 24 m horizontally away from the wall, by what vertical distance on the wall will it miss the target?  
**Answer :** 5 m
- A particle starting from rest travels on a semicircle of radius  $R$ , with uniform tangential acceleration  $a$ . Find the average velocity of the particle and the average acceleration of the particle in the duration of journey.  
**Answer :**  $\langle v \rangle = \sqrt{\frac{2Ra}{\pi}}$ ,  $\langle a \rangle = a$
- An electric motor takes 4 seconds to attain its full speed of 1800 rpm after switching on, and it takes 6 seconds to stop from full speed when it is switched off.

## Motion in Two &amp; Three Dimension

Assuming acceleration and deceleration uniform find the total number of rotations made by motor if it is switched off immediately when it attains full speed.

**Answer :** 150 rev.

12. A particle starting from rest moves on a circle of radius 12 m with uniform tangential acceleration of  $4 \text{ ms}^{-2}$ . Find the total acceleration of the particle 1.5 seconds after the start of journey.

**Answer :**  $5 \text{ ms}^{-2}$

13. A particle is projected with a velocity  $u$  horizontally from an elevated point. Find the radius of curvature of trajectory of a projectile  $t$  seconds after projection.

$$\text{Answer : } R = \frac{(u^2 + g^2 t^2)^{3/2}}{ug}$$

14. A projectile is fired from level ground at an angle  $\theta$  above the horizontal.

a) Show that the elevation angle  $\phi$  of the highest point as seen from the launch point is related to  $\theta$ , the elevation angle of projection, by

$$\tan \phi = \frac{1}{2} \tan \theta.$$

b) Calculate  $\phi$  for  $\theta = 45^\circ$ .

**Answer:**  $\tan^{-1}\left(\frac{1}{2}\right)$

15. A particle projected for maximum range attains maximum height of 20 m. When it is projected with same velocity in such a way that its range is double the maximum height attained, find the range of the particle in the second projection.

**Answer :** 64 m

16. A particle is projected horizontally from a point on the incline plane inclined at an angle of  $\frac{\pi}{4}$  with a velocity  $u$ . How far from the point of projection the particle will strike the plane?

$$\text{Answer : } \frac{2\sqrt{2}u^2}{g}$$

17. The direction of motion of a projectile at a certain instant is inclined at an angle  $\alpha$  to the horizontal.  $t$  seconds later it is inclined an angle  $\beta$  to the horizontal. Find the horizontal component of velocity of projection in terms of  $g$ ,  $t$ ,  $\alpha$  and  $\beta$ .

$$\text{Answer : } \frac{gt}{\tan \alpha - \tan \beta}$$

18. A platform of length 5.5 m and width 2.5 m is moving horizontally along its length.

A toy gun fires a shot horizontally at an angle  $\sin^{-1}\left(\frac{5}{13}\right)$  from the direction of motion of the platform passes over the platform entering at one corner and leaving the diagonally opposite corner. Find the velocity of platform and the gun shot if it takes 2 sec. to pass over the platform.

**Answer :** 0.25 m/s;  $v = 3.25 \text{ m/s}$

19. A truck of width  $w$  is moving with velocity  $u$  along a straight horizontal road. A person starts to cross the road with uniform speed  $v$ , when the truck is a distance  $d$  away from him. For what minimum value of  $v$  he will be able to cross the road.

$$\text{Answer : } v_{\min} = \frac{uw}{\sqrt{d^2 + w^2}}$$

20. If the ratio of times of flight for two projections of an object with the same velocity of projection  $u$  for same range is 2. Find the range of projection.

$$\text{Answer : } \frac{4u^2}{5g}$$

21. A particle is projected with a velocity  $2\sqrt{g\ell}$  so that it passes over two poles of height  $\ell$  each at a horizontal separation of  $2\ell$ , grazing their tops. Find the time it takes to perform the journey between the poles.

**Answer :**  $2\sqrt{\ell/g}$

22. A boy wants to throw a stone passing a pond of width 40 m. He can throw the same stone to a maximum height of 25 m. At what angle of projection he should throw the stone to cross the pond?

**Answer :**  $26.5^\circ \leq \theta \leq 63.5^\circ$

23. Find the radius of curvature of the path of a particle projected with a velocity  $u$  at an angle  $\theta$ , at a point where tangent to the trajectory forms an angle  $\phi$  with the horizontal.

**Answer :**  $\frac{u^2 \cos^2 \theta}{g \cos^3 \phi}$

24. A particle moves on a circle of radius  $R$  with uniform tangential acceleration  $a$ , starting from rest. Find its total acceleration as a function of time.

**Answer :**  $a\sqrt{1 + \frac{a^2 t^4}{R^2}}$

25. For a particular firing from a cannon the range of shell on horizontal ground is  $R$  and the height attained by projectile is  $H$ . What maximum range the shell fired with same velocity from the cannon can attain.

**Answer :**  $2H + \frac{R^2}{8H}$

26. A projectile projected from  $O$  at an elevation  $\alpha$ . After  $t$  seconds its position appeared to have an elevation  $\beta$  as seen from  $O$ . Find the initial velocity of projection of the particle.

**Answer :**  $\frac{gt \cos \beta}{2 \sin(\alpha - \beta)}$

27. A particle is projected with a velocity  $2\sqrt{ag}$  to clear the two walls (perpendicular to plane of projection) of equal height  $a$ , separated by a horizontal distance  $2a$ . Find the angle of projection.

**Answer :**  $\pi/3$

28. If  $h_1$  and  $h_2$  are the greatest heights and  $t_1$  and  $t_2$  the times of flight for the two paths having same range  $R$  and the same initial speed, show that

$$R = 4\sqrt{h_1 h_2} \quad \text{and} \quad R = \frac{gt_1 t_2}{2}.$$

29. A particle projected horizontally from the top of a tower of height 1.8 m strikes the ground target a distance 2.4 m from the base of the tower. If the particle is projected with same velocity directly aiming the same point where it landed on ground, by what distance it will fall short of the target?

**Answer :** 1.1 m

30. A particle is projected horizontally from the top of a tower with a velocity  $12 \text{ ms}^{-1}$ . At the same instant another particle is projected from the bottom of the tower in the same vertical plane at  $30^\circ$  from horizontal. If both the particles strike the horizontal plane simultaneously at the same point, find height of the tower.

**Answer :** 9.6 m

31. A particle moves on a circle with angular velocity varying as

$$\omega = 12t - 3t^2$$

Find average angular velocity of the particle between rest to rest.

**Answer :** 8 rad/s

32. At what angle should a particle be projected with a velocity  $24 \text{ m s}^{-1}$  for just to pass over an obstacle 16 m high at a horizontal distance 32 m?

$$\text{Answer : } \theta = \tan^{-1} \left( \frac{9 \pm \sqrt{11}}{5} \right)$$

33. Two stones are projected simultaneously from a point  $P$  in the same vertical plane with a velocity  $20 \text{ m s}^{-1}$ , one at an angle  $37^\circ$  above horizontal and the other at an angle  $37^\circ$  from vertical. Find the distance between the stones one second after the projection.

$$\text{Answer : } 4\sqrt{2} \text{ m}$$

34. From an elevated point  $A$ , a stone is projected vertically upwards. When the stone reaches a distance  $h$  below  $A$  its velocity is double of what it was at a height  $h$  above  $A$ . Find the greatest height attained by the stone.

$$\text{Answer : } \frac{5h}{3}$$

35. In a projectile motion, at the top most point of its trajectory speed of the projectile is  $\sqrt{\frac{2}{5}}$  times of its speed at half the maximum height attained by the projectile. Find the angle of projection of the projectile.

$$\text{Answer : } \frac{\pi}{3}$$

36. A particle is projected with a velocity  $u$  at angle  $\theta$  from horizontal. Find the magnitude of average velocity of particle in the duration in which it reaches to the highest point of trajectory from the point of projection.

$$\text{Answer: } \left( \frac{u}{2} \right) \sqrt{1 + 3 \cos^2 \theta}$$

37. A projectile projected from  $O$  at an elevation  $\alpha$ . After  $t$  seconds its position appeared to have an elevation  $\beta$  as seen from  $O$ . Find the initial velocity of projection of the particle.

$$\text{Answer : } \frac{gt \cos \beta}{2 \sin(\alpha - \beta)}$$

38. A cyclist moving due east with a velocity  $10 \text{ m/s}$  experiences a wind blowing from north. When he increases his velocity by  $10\sqrt{3} \text{ m/s}$  the wind appears to blow from  $30^\circ$  north of east. Find the true velocity of wind.

$$\text{Answer : } 10\sqrt{2} \text{ m/s, exactly form north-west.}$$

39. A cyclist moving due east with a velocity  $u$  experiences wind blowing from a certain direction with certain magnitude of velocity. When he takes a  $90^\circ$  turn and moves due north with a velocity  $v$ , he experiences wind blowing from exactly opposite direction in his compass with same magnitude of velocity. Find the actual velocity of wind.

$$\text{Answer : } \frac{\sqrt{u^2 + v^2}}{2} \text{ at } \tan^{-1} \left( \frac{v}{u} \right) \text{ north of east.}$$

40. A swimmer takes time  $t_1$  to cross a river of width  $w$  perpendicular to its flow from one bank to other and back. He takes time  $t_2$  to swim the same distant  $w$  down the flow and back. What time will he take to swim a distance  $2w$  in the still water if he maintains same velocity relative to water in all cases?

$$\text{Answer : } \frac{t_1^2}{t_2}$$