MOTION IN TWO & THREE DIMENSION

1. Men are running along a straight horizontal road with uniform speed 15 km/hr at equal spacing of 20 m between them. Cyclists are riding on the adjacent, parallel track with uniform speed 25 km/hr at equal spacing of 30 m between them. Find the velocity with which an observer should move on the same road in opposite direction so that whenever he meets a runner he also meets a cyclist.

Answer : 15 km/hr

2. Two particles are projected simultaneously with the same speed u in the same vertical plane at angles of elevation θ and 2θ, with $\left(\theta < 45^\circ\right)$. At what time will their velocity vectors be parallel?

Answer : $\dfrac{u}{g}\dfrac{\cos\left(\theta/2\right)}{\sin\left(3\theta/2\right)}$

3. A particle is projected from a point at a height $3h$ above a horizontal plane, the direction of projection making an angle θ with the horizontal. Show that if the greatest height above the point of projection is h, the horizontal distance traveled before striking the horizontal plane is

$6h\cot\theta$.

4. A projectile started from O at an elevation α. After t second its position appeared to have an elevation β as seen from O. Prove that initial velocity is

$$\frac{gt\cos\beta}{2\sin(\alpha-\beta)}$$

5. A particle projected vertically upwards, takes time t_1 to reach a height h. If t_2 is the time from this point to the ground again, prove that

a) $h = \frac{1}{2}gt_1t_2$,

b) maximum height $h_{\max} = \dfrac{g(t_1+t_2)^2}{8}$,

c) velocity of the particle at a height $h/2$ is $\dfrac{1}{2}g\sqrt{t_1^2+t_2^2}$.

6. A particle P moves on a circle in x-y plane, with centre O at the origin, and radius $\dfrac{6\sqrt{3}}{\pi}$ m. At time t seconds the radius vector OP has rotated anti clockwise from a fixed position OA along x-axis, through an angle θ radian where $\theta = \dfrac{\pi}{3}\sin 2t$. Find the speed, velocity, and the magnitude of acceleration when $t = \pi/12$.

Answer: $6\ \text{m s}^{-1},\ 3\left(-\hat{\mathbf{i}}+\sqrt{3}\,\hat{\mathbf{j}}\right)\text{m s}^{-1},\ -\sqrt{3}\pi\left(\sqrt{3}\,\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)\ \text{m s}^{-2}$

7. The position of a particle as a function of time is

$$\vec{\mathbf{r}} = (4\sin 2\pi t)\hat{\mathbf{i}} + (4\cos 2\pi t)\hat{\mathbf{j}}$$

where r is in meters and t in seconds.

a) Show that the path of this particle is a circle of radius 4 m with its centre at the origin.

b) Calculate the velocity vector. Show that $\dfrac{v_x}{v_y} = -\dfrac{y}{x}$

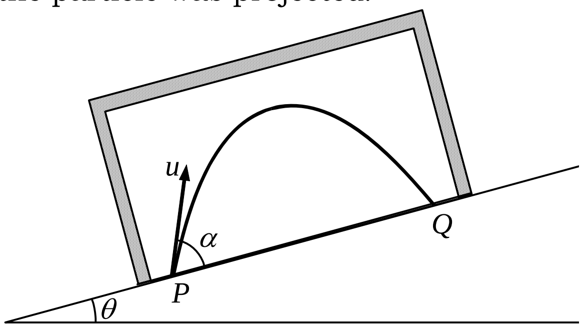
c) Calculate the acceleration of the particle.

Answer: b) $\vec{\mathbf{v}} = 8\pi\left(\cos 2\pi t\,\hat{\mathbf{i}} - \sin 2\pi t\,\hat{\mathbf{j}}\right)$; **c)** $\vec{\mathbf{a}} = -16\pi^2\left(\sin 2\pi t\,\hat{\mathbf{i}} + \cos 2\pi t\,\hat{\mathbf{j}}\right)$

8. A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in the figure.

a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.



Answer: a) $\dfrac{u^2 \sin 2\alpha}{g\cos\theta}$; **b)** $\dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

9. A particle revolves about a stationary axis following the relation $\varphi = at - bt^3$ where φ is the angular displacement of particle. Find the average angular velocity of particle from $t = 0$ to its complete stop. Also find the angular acceleration of the particle at the instant it stops.

Answer: $\dfrac{2a}{3}$, $-2\sqrt{3ab}$

10. A particle revolves around a stationary axis with its angular velocity ω following the relation $\omega = \omega_0 - a\varphi$ where φ is the angular displacement of particle. Find the angular displacement of the particle as function of time.

Answer: $\varphi = \dfrac{\omega_0}{a}\left(1 - e^{-at}\right)$

11. A particle starting from origin of coordinates, moves in the x-y plane with velocity $\vec{\mathbf{v}} = a\hat{\mathbf{i}} + bx\hat{\mathbf{j}}$ where a and b are positive constants.

a) find the equation of trajectory of the particle,

b) find the total tangential and normal acceleration of particle.

Answer: a) $x^2 = \dfrac{2a}{b}y$; **b)** ab, $\dfrac{ab^2t}{\sqrt{1+b^2t^2}}$, $\dfrac{ab}{\sqrt{1+b^2t^2}}$

12. Two particles are projected with same velocity $5\sqrt{3}$ ms^{-1} from the top of a tower of height 10 m. First particle is projected at an angle 60^o above horizontal, after what time the second particle be projected horizontally so that they collide in air. Find the coordinates of the point of collision with respect to point of projection.

Answer: 1sec, $\left(5\sqrt{3}, 5\right)$

13. An object starts moving from a point (3, $5/4$) with uniform acceleration $3/2$ ms^{-2} along x-axis, in x-y plane with x-axis horizontal and y-axis vertical. At the same instant a particle is projected in the same vertical plane from (0, 0). If the particle strikes the object in its downward journey at an angle $\pi/4$ from horizontal, find the velocity of projection of particle and the time it takes to strike the object after projection. (g = 10 ms^{-2})

Answer: 7.3 ms^{-1} at $\tan^{-1}\left(5/3\right)$, 1 sec.

14. A batsman at the origin of coordinates in a cricket ground hits a ball which moves with constant velocity $\left(7.5\hat{\mathbf{i}}+10\hat{\mathbf{j}}\right)$ on the horizontal ground. A fielder at the position $\left(46\hat{\mathbf{i}}+28\hat{\mathbf{j}}\right)$ starts running with a constant velocity 5 ms^{-1} immediately after the ball is hit. Find the shortest time in which he can intercept the ball.

Answer: 4 sec.

15. Wind is blowing in the line of two parallel railway tracks with speed w. Two trains moving with same speed in the opposite directions on these tracks have steam tracks with length of one double that of the other. Find the speed of each train.

Answer: $3w$

16. In a wind blowing from south with constant speed u a helicopter flies horizontally with constant velocity in a direction θ east of north form a point A to a point B. $AB = c$. The speed of helicopter relative to air is λu, where $\lambda > 1$. Find the speed of helicopter along AB. The helicopter returns from B to A with constant velocity and same speed λu relative to air and in the same wind. Prove that the total time for the journeys is

$$v = u\cos\theta + u\sqrt{\lambda^2 - \sin^2\theta};\quad t = \frac{2c\sqrt{\lambda^2-\sin^2\theta}}{u\left(\lambda^2-1\right)}$$

17. At $t = 0$, the engine of a boat moving with a velocity v_0 is switched off, due to which the boat starts decelerating with magnitude of deceleration proportional to instantaneous speed of the boat $\left(|a| = kv\right)$, where k is a constant. What time will it take for its velocity to become $v_0/2$, what distance will it travel before it comes to complete stop?

Answer: $\frac{\ln 2}{k}, \frac{v_0}{k}$

18. At $t = 0$, two points A and B on a straight line have position, velocity and acceleration respectively as $\vec{\mathbf{r}}_A = 0\hat{\mathbf{i}}$ m, $\vec{\mathbf{r}}_B = 100\hat{\mathbf{i}}$ m, $\vec{\mathbf{v}}_A = 10\hat{\mathbf{i}}$ m s^{-1}, $\vec{\mathbf{v}}_B = -20\hat{\mathbf{i}}$ m s^{-1}, $\vec{\mathbf{a}}_A = -3\hat{\mathbf{i}}$ m s^{-2} and $\vec{\mathbf{a}}_B = b\hat{\mathbf{i}}$ m s^{-2}. For what value of b collision between A and B will be just avoided.

Answer: $b = 3/2$

19. A particle travels with uniform tangential acceleration on a circular arc of radius R. Its velocity at P is u and when it reaches the diametrically opposite point Q its velocity becomes $2u$. Find the magnitude of average velocity and the magnitude of average linear acceleration of the particle between P and Q.

Answer: $\frac{3u}{\pi}, \frac{9u^2}{2\pi R}$

20. A particle moving with a velocity v_0 at $t = 0$, starts experiencing a deceleration whose magnitude is directly proportional to the n^{th} power ($n < 1$) of the magnitude of its instantaneous velocity. Find the average velocity of the particle in the duration it completely stops.

Answer: $\langle v\rangle = \left(\frac{1-n}{2-n}\right)v_0$

21. A particle is moving in x-y plane according to the law $x = 2a\sin\omega t$ and $y = a\cos\left(\omega t - \pi/6\right)$ where a and ω are positive constants. Write the equation of the locus. Find angle between velocity and acceleration vectors at $t = 0$.

Answer: $x^2 + 4y^2 - 2xy - 3a^2 = 0$, $\tan^{-1}(-4)$

22. A trolley moves down an incline with angle of inclination 30°, starting from rest with a uniform acceleration 10 ms^{-2}. 2 seconds after the start a boy sitting in the trolley throws a ball with a velocity 20 ms^{-1} w.r.t. himself in the same vertical plane in which trolley is moving at such an angle that he is able to catch the ball again. Neglecting air resistance, find the time of flight of the ball and the maximum distance of the ball from the boy in the trolley.

Answer: 4 second, 20 m

23. A swimmer swims perpendicular to river flow of a 97.5 m wide river flowing with a uniform rate of 4 ms^{-1} along its entire width. The initial velocity of swimmer is 8 ms^{-1} relative to water, which decreases at a constant rate of 0.2 ms^{-2}. What time will it take to reach the other bank of the river and the distance swimmer drifts in this time along the river flow?

Answer: 15 s, 60 m

24. A particle is moving in circular path of radius R, with a tangential acceleration $a_t = -kv$, where v is its instantaneous speed and k is a positive constant. The initial speed of the particle is $v_o = 2kR$. Find the angle between instantaneous velocity vector and total instantaneous acceleration vector as a function of time.

Answer: $\theta = \pi - \tan^{-1}\left(2e^{-kt}\right)$

25. A swimmer can swim with a velocity 2 ms^{-1} in still water and can walk along the bank with a velocity 4 ms^{-1}. He enters at a point A on one bank of a river of width 200 m flowing with a uniform velocity 4 ms^{-1} along its entire width. The swimmer reaches the other bank at point C swimming at a constant angle from perpendicular to river flow, and then walks to the point B exactly opposite to the point A on the other bank of the river. Find the angle from the perpendicular to river flow at which the swimmer should swim for the time of entire journey to be minimum.

Answer: $\sin^{-1}\left(1/4\right)$ opposite to river flow.

26. A particle is moving with a constant speed v along the trajectory $y = ax^2 + x$, in the direction of increasing x coordinate where a is a positive constant. Find the radius of curvature and magnitude of acceleration at origin of coordinates.

Answer: $\dfrac{\sqrt{2}}{a}$; $\sqrt{2}av^2$

27. A cyclist moving horizontally with a velocity 10 ms^{-1} experiences vertical rain. On return journey with the speed 20 ms^{-1}, he finds that the rain drops strike him at an angle 30° with the horizontal. Find the true velocity of rain drops.

Answer: 20 ms^{-1}

28. A particle is projected from a point O with velocity v at an angle α to the horizontal. The horizontal range through O is R. If the distance between the two points on the trajectory which the direction of motion of the particle makes an angle $\alpha/2$ with the horizontal is $R/3$, find the value of α.

Answer: $\pi/3$

29. A is a point on a smooth plane inclined at an angle α to the horizontal, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors passing through A, $\hat{\mathbf{i}}$ in the horizontal plane and $\hat{\mathbf{j}}$ in the line of greatest slope. A point C has position $x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ relative to A. A particle is projected from A in the face of the plane with a speed v at a variable angle β to $\hat{\mathbf{i}}$. If it can only just reach C show that

$$x(\tan\beta - \cot\beta) = 2y$$

30. The line joining two points A, B is of constant length a and the velocities of A and B are in direction which make angle α and β respectively with AB. Prove that the angular velocity of AB is

$$\frac{u\sin(\alpha - \beta)}{a\cos\beta}$$

where u is the velocity of A.

31. If the times of flight for the two paths corresponding to two possible direction of projection to get a given range for same velocity of projection are in the ratio 2 : 1, find the ratio of this range and the maximum possible range for same velocity of projection.

Answer: 0.8

32. Two parallel lines in the same vertical plane are each inclined to the horizontal at an angle α. From a point midway between them a particle is projected so as to graze one line and fall perpendicularly on the other. Find the angle that the direction of projection makes with either line.

Answer: $\tan^{-1}\{(\sqrt{2}-1)\cot\alpha\}$

33. A person in a boat throws a stone at an elevation α and with a velocity V relative to the boat which is moving with a velocity v in the direction of the object aimed at. If the ratio v/V is small, find the small alteration in the elevation necessitated by the motion of a boat for a given horizontal range.

Answer: $\delta\alpha = -\dfrac{v\sin\alpha}{(V + v\cos\alpha)}$

34. With what minimum speed must a particle be projected from origin so that it is able to pass through a given point A (a, b)?

Answer: $v \geq g\sqrt{b + \sqrt{a^2 + b^2}}$

35. A perfectly elastic particle is projected with a velocity V in a vertical plane through the line of greatest slope of an inclined plane of elevation θ. If after striking the plane it rebounds vertically, find the time it will take to return to the point of projection.

Answer : $\dfrac{6V}{g\sqrt{1 + 8\sin^2\theta}}$

36. A particle is projected with a velocity $u = \sqrt{2ag}$ towards a vertical wall perpendicular to plane of projectile, at a horizontal distance a from the point of projection. Find the maximum height it can reach on the wall?

Answer : $\dfrac{3a}{4}$

37. If two particles be projected in the same vertical plane with velocity u_1 and u_2 and angle of projection α_1 and α_2 with the horizontal, after what time the second particle be projected to strike the first particle in route? Also find the time that elapses when their velocities are parallel.

Answer : $\dfrac{2}{g}\dfrac{u_1u_2\sin(\alpha_1 - \alpha_2)}{(u_1\cos\alpha_1 + u_2\cos\alpha_2)}$

38. A ball is projected at an angle α to the horizon so as to just clear two walls of equal height a at a horizontal separation $2a$ from each other. Show that the range of ball is equal to

Answer : $2a\cot\left(\alpha/2\right)$

39. A particle is projected from a point exactly midway between two straight lines in same vertical plane inclined at 45° from horizontal, in such a way that it grazes the upper line and crosses the lower line perpendicular to it. Find the angle of projection of the particle.

Answer : $\frac{3\pi}{8}$

40. A particle is projected from a point on horizontal ground to clear a wall of height h at a horizontal distance a from the point of projection. Find the minimum velocity of projection for this.

Answer : $u_{min} = \sqrt{g\left(h + \sqrt{h^2 + a^2}\right)}$

Further suggested readings :

1. H C Verma (for Problems only Objective & Subjective)
2. D C Pandey IIT Objective
3. I E Irodov (1.1 - 1.50)