KINEMATICS

Motion in one dimension

1. Basic definitions

a) Displacement $\overrightarrow{AB} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \vec{\mathbf{r}}$

b) Average velocity $\langle \vec{\mathbf{v}} \rangle = \dfrac{\Delta \vec{\mathbf{s}}}{\Delta t}$

average speed when two

equal distances traveled with speed v_1 and v_2, $\langle v \rangle = \dfrac{2v_1 v_2}{v_1 + v_2}$

distances traveled for equal time with speed v_1 and v_2, $\langle v \rangle = \dfrac{v_1 + v_2}{2}$

c) Average acceleration $\langle \vec{\mathbf{a}} \rangle = \dfrac{\Delta \vec{\mathbf{v}}}{\Delta t} = \dfrac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t}$

2. Equations of motion (valid only for uniform acceleration)

a) $v = u + at$

b) $s = ut + \dfrac{1}{2}at^2$

c) $v^2 = u^2 + 2as$

d) $\langle v \rangle = \dfrac{u + v}{2}$

e) $s_{n^{th}} = u + \dfrac{1}{2}a(2n - 1)$

3. Graphs

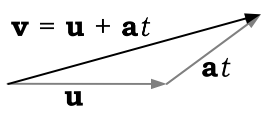
a) area under velocity-time graph gives total displacement
b) slope of tangent in velocity-time graph gives instantaneous acceleration
c) slope of tangent in displacement-time graph gives instantaneous velocity
d) area under acceleration-time graph gives change in velocity
e) distance-time graph is a monotonically increasing graph and when displacement-time graph goes down its mirror image about horizontal line gives distance time graph

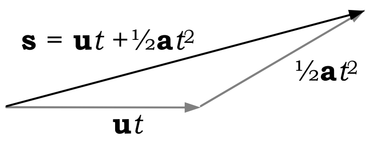
4. Vector form of equations of motion

a) $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

b) $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\,\mathbf{a}t^2$

c) $\mathbf{v}.\mathbf{v} = \mathbf{u}.\mathbf{u} + 2\mathbf{a}.\mathbf{s}$ (here all the three terms are scalar)



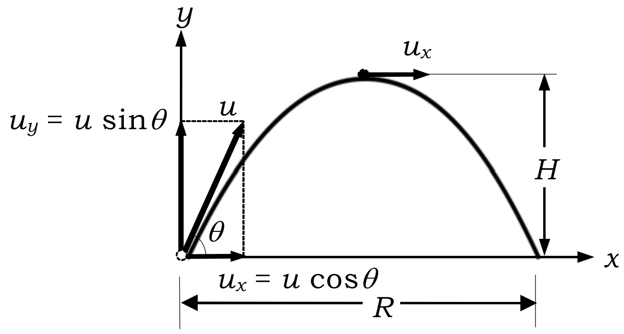


5. Whether initial velocity is zero or final velocity is zero the shape of equations of motion remain same

If initial velocity is zero, then the equation of motion can be written as: $v = at$ $s = \frac{1}{2}at^2$ $v^2 = 2as$ $\langle v \rangle = \frac{v}{2}$	The same is true if final velocity is zero and initial velocities is u. (thus then the acceleration = $-a$) $v = 0 = u - at \quad \Rightarrow \quad u = at$ $s = ut - \frac{1}{2}at^2 = at^2 - \frac{1}{2}at^2 \Rightarrow \quad s = \frac{1}{2}at^2$ $v^2 = 0 = u^2 - 2as \quad \Rightarrow \quad u^2 = 2as$ $\langle v \rangle = \frac{0+u}{2} = \frac{u}{2} \quad \Rightarrow \quad \langle v \rangle = \frac{u}{2}$

Motion in Two dimensions

1. Projectile (on horizontal plane)



a) Time of flight $T = \frac{2u\sin\theta}{g}$

b) Range of projectile $R = \frac{u^2 \sin 2\theta}{g}$

$R_{\max} = u^2/g$ when projected at an angle $\theta = \pi/4$

c) Height of projectile $H = \frac{u^2 \sin^2\theta}{2g}$

$H_{\max} = u^2/2g$ when projected at an angle $\theta = \pi/2$

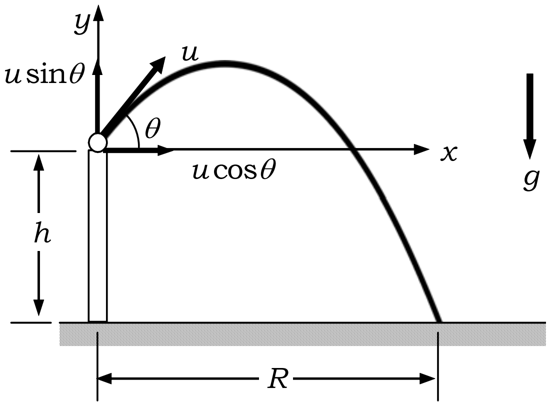
d) $R_{(\pi/2 - \theta)} = R_\theta$

e) If the particle is projected at $\theta = \tan^{-1}(4)$ then $R = H$

f) The expression $y = (\tan\theta)x - \left(\frac{g}{2u^2}\sec^2\theta\right)x^2$ is called equation of trajectory

The two roots of this equation represent those x-positions where y is same.

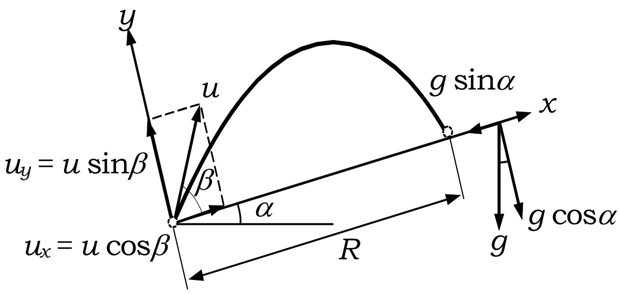
2. Projectile from an elevated point



a) time of journey $t = \dfrac{\pm u\sin\theta + \sqrt{u^2\sin^2\theta + 2gh}}{g}$

b) Range on horizontal plane $R = u\cos\theta\left\{\dfrac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2gh}}{g}\right\}$

3. Projectile on inclined plane



a) Time of flight $T = \dfrac{2u\sin\beta}{g\cos\alpha}$

b) Range along the incline $R = \dfrac{2u^2\sin\beta\cos(\alpha+\beta)}{g\cos^2\alpha}$

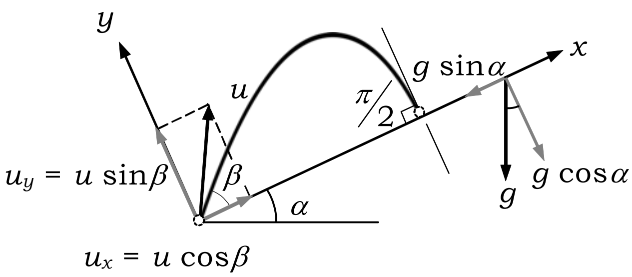
c) For range to be maximum particle must be projected along the angular bisector of angle between plane and incline.

$\alpha + 2\beta = \pi/2$

d) Maximum range up the incline $R_{max} = \dfrac{u^2}{g(1+\sin\alpha)}$

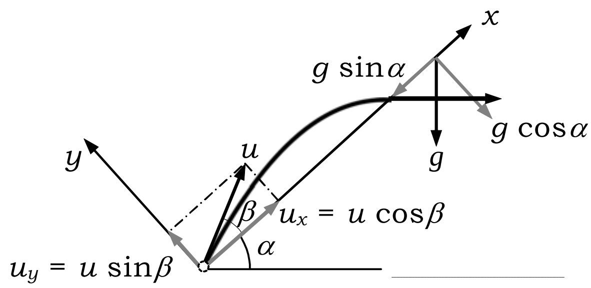
e) Maximum range down the incline $R_{max} = \dfrac{u^2}{g(1-\sin\alpha)}$

f) Condition for the object to strike the incline plane perpendicular to it



$\tan\alpha \tan\beta = \frac{1}{2}$

g) Condition for the object to strike the incline plane horizontally



$$\frac{\sin\beta}{\cos\alpha \sin(\alpha+\beta)} = \frac{1}{2}$$

4. Circular motion

a) Angular displacement $\theta = s/R$

b) Fundamental equations

i) average angular velocity $\langle\omega\rangle = \theta/t$

ii) instantaneous angular velocity $\omega = \frac{d\theta}{dt}$

iii) instantaneous angular acceleration $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$

c) Equations of motion (valid only for uniform angular acceleration)

i) $\omega_f = \omega_i + \alpha t$

ii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

iii) $\omega_f^{\,2} = \omega_i^{\,2} + 2\alpha\theta$

iv) $\theta_{n^{th}} = \omega_i + \frac{\alpha}{2}(2n-1)$

v) $\langle\omega\rangle = \frac{\omega_i + \omega_f}{2}$

d) Relation between linear and angular quantities

i) $s = R\theta$

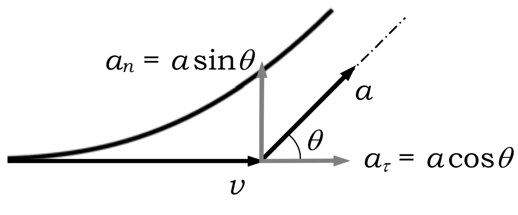
ii) $v = R\omega$

iii) $a = R\alpha$

e) Component of instantaneous acceleration along the instantaneous velocity is tangential acceleration and component of instantaneous acceleration perpendicular to instantaneous velocity is normal acceleration.

$$a_\tau = \frac{d}{dt}|\vec{\mathbf{v}}|$$

$$a_n = \frac{v^2}{R}$$



Relative velocity

1. $\vec{\mathbf{v}}_{rel} = \vec{\mathbf{v}}_{object} - \vec{\mathbf{v}}_{observr}$ or

$\vec{\mathbf{v}}_{object} = \vec{\mathbf{v}}_{rel} + \vec{\mathbf{v}}_{observr}$

2. River swimmer problem

a) Time taken to cross the river is always calculated by component of swimmer's velocity perpendicular to river flow

$$\tau = \frac{l}{v\cos\alpha}$$

b) To cross the river in minimum time one should swim perpendicular to river flow

$$\tau_{min} = \frac{l}{v}$$

c) Crossing the river in minimum distant

when v (swimmer velocity) > u (river flow velocity)

$\sin\alpha = u/v$,

drift = 0 and

$$\tau = \frac{d}{\sqrt{v^2 - u^2}}$$

when v (swimmer velocity) < u (river flow velocity)

$\sin\alpha = v/u$,

drift $s_{min} = \dfrac{d\sqrt{u^2 - v^2}}{v}$ and

$$\tau = \frac{ud}{v\sqrt{u^2 - v^2}}$$

Facts :

1. Distance traveled from A to B is always greater than or at least equal to the displacement of object from A to B.

2. Magnitude of instantaneous velocity is equal to instantaneous speed but magnitude of average velocity is less than the average speed for object moving with non-uniform velocity and magnitude of average velocity is equal to the average speed for object moving with uniform velocity.

3. If $v \propto t$ or $v \propto \sqrt{s}$ acceleration of the object is uniform.