

MAGNETISM

- A circular coil of radius 8.8 cm has 10 turns. Find the magnetic induction at the centre produced by a current of 1.0 A passing through it. Find the resultant field at its centre when the plane of coil is vertical and

 - in the magnetic meridian
 - perpendicular to magnetic meridian.

The horizontal component of the earth's magnetic induction $\vec{B}_H = 40 \mu\text{T}$

Answer : 30 μT ; a) 50 μT ; b) $(40 \pm 30) \mu\text{T}$

- A charge q moves in a circular path of radius r with a speed v . Calculate the induction of magnetic field produced at the centre of circle.

$$\text{Answer : } B = \frac{\mu_0 qv}{4\pi r^2}$$

- Two long, straight, horizontal parallel wires, one above the other, separated by a distance $2a$. If the wires carry equal currents i in (A) opposite directions, (B) in same direction what is the magnitude of magnetic field in the plane of the wires at a point

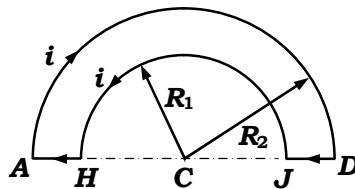
 - half way between them
 - at a distance a above the upper wire?

$$\text{Answer : a) (A) } B = \frac{\mu_0 i}{\pi a}, \text{ (B) } B = 0; \text{ b) (A) } B = \frac{\mu_0 i}{3\pi a}, \text{ (B) } B = \frac{2\mu_0 i}{3\pi a}$$

- You are given a length ℓ of wire in which a current i may be established. The wire may be formed into a circle or a square. Find the value of magnetic induction B at the center in the two cases.

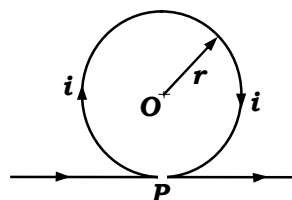
$$\text{Answer : } B_{\text{circle}} = \frac{\pi\mu_0 i}{\ell}, B_{\text{square}} = \frac{8\sqrt{2}\mu_0 i}{\pi\ell}$$

- Use Biot-Savart law to calculate the magnetic induction B at C , the common centre of semicircle arcs AD and HJ of radii R_2 and R_1 respectively, forming part of circuit $ADJHA$ carrying current i , as shown in the adjacent diagram.



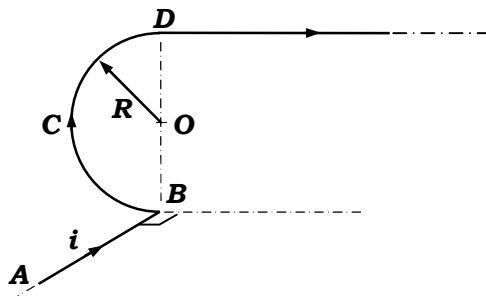
$$\text{Answer : } B = \frac{\mu_0 i (R_2 - R_1)}{4R_2 R_1}$$

- A long wire is bent as shown in the figure. What will be the magnitude and direction of the field at the centre O of circular portion, if a current i is passed through the wire? Assume that the various portions of the wire do not touch at point P .



Answer : $B = \frac{\mu_0 i}{2r} \left(1 - \frac{1}{\pi} \right)$

7. Find magnetic field at the point O, in the following figure in which ABC is a semicircle with centre at O and the wire AB is perpendicular to plane of rest of the figure.



Answer : $\frac{\mu_0 i}{4\pi R} \sqrt{\pi^2 + 2(1 + \pi)}$

8. A thin straight non-conducting rod AB of length ℓ has a total charge Q distributed along its length such that the linear charge density $\lambda \propto x$ (x is the distance from its end A). The rod rotates with an angular velocity ω about an axis perpendicular to its length passing through its end A. Find the strength of magnetic field at A. also find the total magnetic moment associated with the system.

Answer : $B = \frac{\mu_0 Q \omega}{2\pi \ell}$; $M = \frac{Q \omega \ell^2}{4}$

9. A source of constant emf E is connected across a cylindrical conductor of length ℓ and radius R. The resistivity of the material of conductor varies with radius r from axis of rod as $\rho = \frac{\rho_0}{(1 - r/R)}$. Find the current density as a function r, hence by

applying Ampere's law find the magnetic field strength inside and outside the rod. Draw schematic plot of the calculated magnetic field strength.

Answer : $j = \frac{E}{\rho_0 \ell} (1 - r/R)$; $B_{inner} = \frac{\mu_0 E}{6\rho_0 \ell R} r(3R - 2r)$; $B_{outer} = \frac{\mu_0 E R^2}{6\rho_0 \ell r}$

10. In figure 1) $AB = \sqrt{3}R$, $BC = 2R$ and $CD = \sqrt{3}R$ respectively are the three sides of rectangle and DEA is a semicircle of radius R. In figure 2) ABC is an equilateral triangle of side ℓ . The two sections of wire at A and C are of infinite length. Find the magnetic field at the point O in the wire frames of all the three figures shown below.

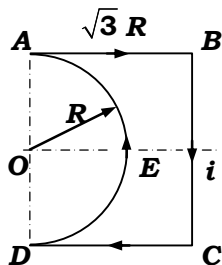


figure a)

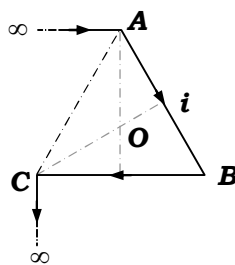


figure b)

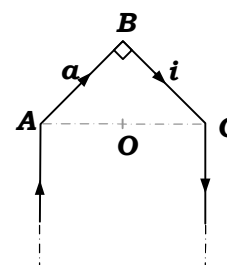


figure c)

Answer : a) $\frac{\mu_0 i}{4R} \left\{ \frac{4}{\sqrt{3}\pi} - 1 \right\}$; b) $\frac{\mu_0 i}{4\pi \ell} \left\{ \sqrt{3} + 11 \right\}$; c) $3\sqrt{2} \frac{\mu_0 i}{2\pi a}$

11. A right circular cone of height $\sqrt{3}a$ and base radius a, charged uniformly on its lateral surface with charge density σ is rotating about its axis with angular velocity

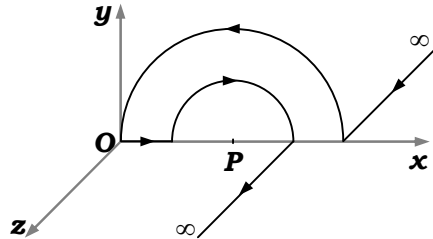
ω . Find the magnetic field at the vertex of the cone. Also find the total magnetic moment associated with the system.

$$\text{Answer : } B = \frac{\mu_0 Q \omega}{16\pi a}; M = \frac{Q \omega a^2}{4}$$

12. A point charge q is moving with a velocity $\vec{v} = v_0(\hat{i} + 2\hat{j})$. At a particular instant it is crossing the origin. Find the magnetic field at the position vector $\vec{r} = a(\hat{i} + \hat{j} + \hat{k})$ at that instant.

$$\text{Answer : } \vec{B} = \frac{\mu_0 q v_0}{12\sqrt{3}\pi a^2} (2\hat{i} - \hat{j} - \hat{k})$$

13. A conductor carrying current i has two semicircular portions in x - y plane as shown in figure. Two straight sections parallel to z -axis are of very large length. If the radius of inner semicircle is a and that of the outer semicircle is $2a$ than find magnetic field at the centre of curvature of the semicircles.

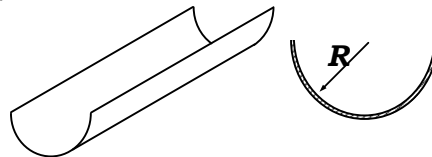


$$\text{Answer : } B = \frac{\mu_0 i}{8a} \sqrt{1 + \frac{9}{\pi^2}}$$

14. Find the magnetic field at the centre of a rectangular wire frame of sides a and b carrying a current i . Also find its value in terms of length of diagonal d and angle between the diagonals φ .

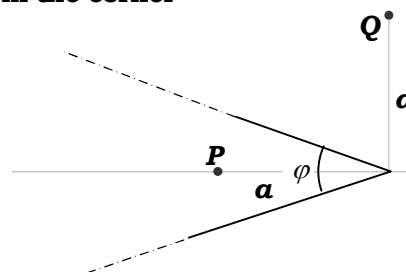
$$\text{Answer : } B = \frac{2\mu_0 i \sqrt{a^2 + b^2}}{\pi ab}; B = \frac{4\mu_0 i}{\pi d \sin \varphi}$$

15. A current i flows in a long straight semi-circular pipe, with cross-section having the form of a thin half-ring of radius R . Find the magnetic field at the centre of curvature of the figure.



$$\text{Answer : } B = \frac{\mu_0 i}{\pi^2 R}$$

16. A long wire carrying current i is bent to form a plane angle α . Find the magnetic field B , a distance a from the corner



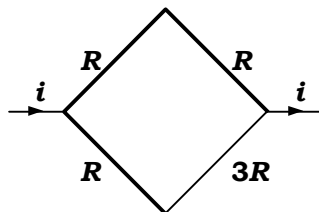
- a) at a point P on the angular bisector
b) at a point Q normally above the plane of figure.

Answer : a) $B = \frac{\mu_0 i}{4\pi a} \cot(\alpha/4)$; b) $B = \frac{\mu_0 i}{2\pi a} \sin(\alpha/2)$

17. A circular coil of radius R having n turns carries a current i . At what distance from the centre on the axis of coil, magnetic field decreases by a factor η ($\eta > 1$) of its value at the centre?

Answer : $x = R\sqrt{\eta^{2/3} - 1}$

18. A square frame is formed by four wires of same length a , of which three of them are of resistance R and one of resistance $3R$ as shown in figure. If a current i enters at one corner and leaves the frame at diagonally opposite corner. Find the magnetic field at the centre of frame.

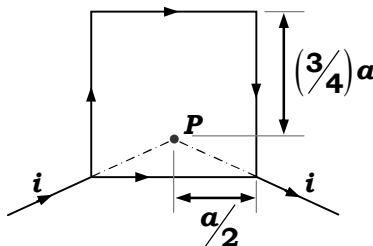


Answer : $\frac{\sqrt{2}\mu_0 i}{3\pi a}$

19. Two large current carrying parallel wires carrying current i , separated by a distance a , are placed perpendicular to plane of the paper. Find the magnetic field at a point equidistant from the two wires at a distance b from each wire in the plane of the paper, if wires carry current
- in the same direction
 - in the opposite direction

Answer : a) $\frac{\mu_0 i}{2\pi b^2} \sqrt{4b^2 - a^2}$; b) $\frac{\mu_0 i}{2\pi b^2} a$

20. A square frame is formed by four wires of resistance R and length a each. A current i enters at one corner and leaves the frame at the adjacent corner as shown in figure. Find the magnetic field at the point P .

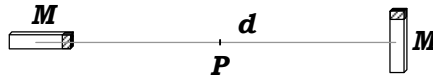


Answer : $\frac{\mu_0 i}{4\pi a} \left(\frac{\sqrt{13}}{3} - \frac{11}{\sqrt{5}} \right)$

21. A long straight wire is carrying a current i . Magnetic field at a point P , a perpendicular distance d from the wire is B_0 . When wire is reduced to finite length $2l$ with P at a distance d on the perpendicular bisector of the wire the magnetic field at P becomes B . Find d/l if $B/B_0 = \eta$.

Answer : $\frac{\sqrt{1-\eta^2}}{\eta}$

22. Figure shows two small identical bar magnets of magnetic moment M each placed at a separation d , with their axes perpendicular to each other. Find the magnetic field at the point P midway between the two.



$$\text{Answer : } B = \frac{2\sqrt{5}\mu_0 M}{\pi d^3}$$

23. A bar magnet of length 8 cm having a pole strength of 1 Am is placed vertically on a horizontal table with its south pole on the table. A neutral point is found on the table at a distance of 6 cm north of the magnet. Find the horizontal component of earth's magnetic field.

$$\text{Answer : } 22 \mu\text{T}$$

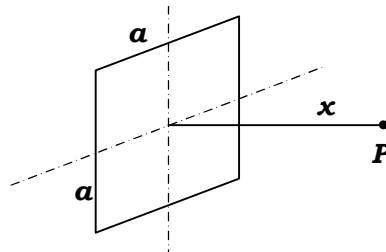
24. A magnetic needle free to rotate in a fixed vertical plane stays in a direction making an angle of 60° with the horizontal. If the dip at that place is 37° , find the angle of the fixed vertical plane with that meridian.

$$\text{Answer : } \theta = \cos^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

25. A magnetic dipole of magnetic moment $0.72\sqrt{2} \text{ Am}^2$ is placed horizontally with the north-pole pointing towards east. Find the position of the neutral point if the horizontal component of earth's magnetic field is $18 \mu\text{T}$.

$$\text{Answer : } 20 \text{ cm from dipole at } \tan^{-1}\sqrt{2} \text{ south of east.}$$

26. A square wire frame of side a carries a current i . Find magnetic field at a point P on its axis at a distance x from its centre as shown in figure.



$$\text{Answer : } \frac{\mu_0 i a^2}{2\pi \left(x^2 + \frac{a^2}{4}\right) \sqrt{x^2 + \frac{a^2}{2}}}$$

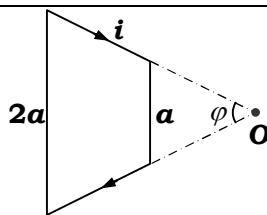
27. A thin flat, infinitely long strip of width w carries a uniformly distributed current i . Determine the magnetic field at a point P in the plane of the strip at a distance x from the near edge of the strip.

$$\text{Answer : } B = \frac{\mu_0 i}{2\pi w} \ln\left(1 + \frac{w}{x}\right)$$

28. A very long rectilinear cylindrical beam of circular cross-section of radius a , having a constant volume charge density ρ , moving along its axis with a velocity v . Find the magnetic field at a distance r from its axis for, $r < a$ and $r > a$.

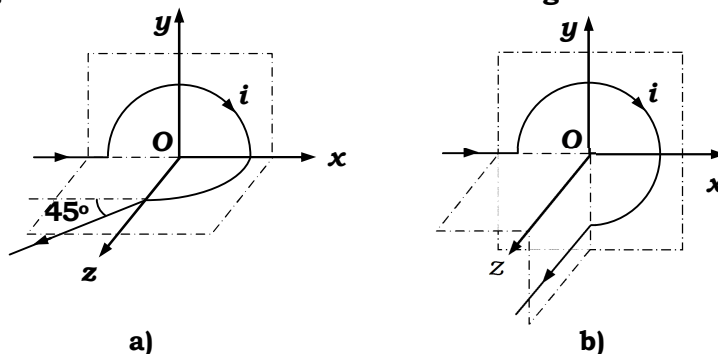
$$\text{Answer : } \vec{B} = \frac{\mu_0 \rho}{2} (\vec{v} \times \vec{r}) \text{ for } r < a, \quad \vec{B} = \frac{\mu_0 \rho a^2}{2r^2} (\vec{v} \times \vec{r}) \text{ for } r > a$$

29. A current i flows in a wire frame in the form of an isosceles trapezium with parallel sides a and $2a$ as shown in figure. Find magnetic field at the point O .



$$\text{Answer : } B = \frac{\mu_0 i \sin^2(\phi/2)}{\pi a \cos(\phi/2)}$$

30. Find magnetic field at point O, if the wire carrying current i has the shape as shown in figure with radius of circular sections being a .



$$\text{Answer : a) } B_o = -\frac{\mu_0 i}{4a} \left(\left(\frac{\sqrt{2}-1}{\pi} \right) \hat{j} + \hat{k} \right); \text{ b) } B_o = -\frac{\mu_0 i}{4a} \left(\frac{1}{\pi} \hat{i} + \frac{3}{2} \hat{k} \right)$$

31. Two long parallel wires carries a constant current i along the same direction. The wires are separated by a distance $2l$. Find the maximum magnitude of magnetic field in the symmetry plane of this system located between the wires.

$$\text{Answer : } \frac{\mu_0 i}{2\pi l}$$

32. A disc of radius R rotates at an angular velocity ω about the axis perpendicular to its surface and passing through its centre. If the disc has a uniform surface charge density σ , find the magnetic induction on the axis of rotation at a distance x from the centre.

$$\text{Answer : } B = \frac{\mu_0 \sigma \omega}{2} \left\{ \frac{R^2 + 2x^2}{\sqrt{R^2 + x^2}} - 2x \right\}$$

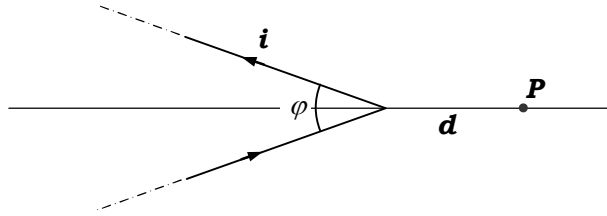
33. A long straight wire of radius r carries a current i uniformly distributed in its cross-section, is placed horizontally in a uniform vertical magnetic field B_0 pointing upwards (earth's magnetic field is to be neglected). Find the maximum and minimum magnitude of resultant magnetic field, and the points where it is maximum or minimum.

$$\text{Answer : } B_{max} = B_0 + \frac{\mu_0 i}{2\pi r} \text{ in both the case at } x = r;$$

$$B_{min} = B_0 - \frac{\mu_0 i}{2\pi r} \text{ if } B_0 > \frac{\mu_0 i}{2\pi r} \text{ at } x = -r;$$

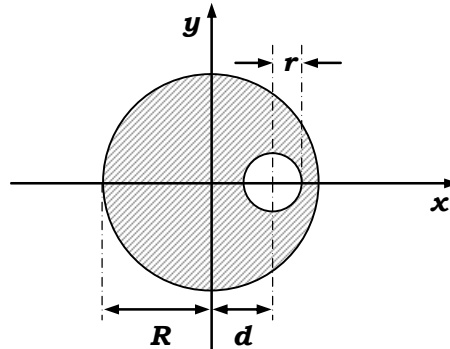
$$B_{min} = 0 \text{ if } B_0 < \frac{\mu_0 i}{2\pi r} \text{ at } x = -\frac{\mu_0 i}{2\pi B_0} \text{ and } x = -\sqrt{\frac{2\pi B_0}{\mu_0 i R}};$$

34. A long wire carrying current i is bent to form a plane angle α . Find the magnetic field B at a point P , a distance d from the corner on the angular bisector as shown in figure.



$$\text{Answer : } B = \frac{\mu_0 i}{2\pi d} \tan\left(\frac{\phi}{4}\right)$$

35. A very long straight cylindrical conductor of radius R having a cylindrical hole of radius r ($r < R$) as shown in figure, is placed with its axis along z -axis carries a current I in the remaining cross-section along z -axis. The axis of cylinder and hole are parallel and distance between them is d . Find the magnetic field on the



- a) x -axis at $x = 2R$
b) y -axis at $y = 2R$.

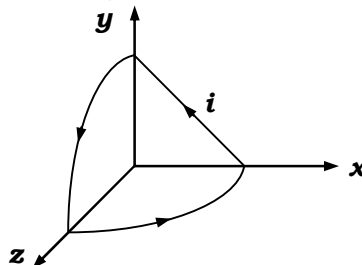
$$\text{Answer : a) } \vec{B} = \frac{\mu_0}{2\pi} \frac{I}{(R^2 - r^2)} \left\{ \frac{R}{2} - \frac{r^2}{(2R - d)} \right\} \hat{j};$$

$$\text{b) } \vec{B} = -\frac{\mu_0 I R}{\pi(R^2 - r^2)} \left\{ \frac{1}{4} - \frac{r^2}{(d^2 + 4R^2)} \right\} \hat{i} + \frac{\mu_0 I r^2 d}{2\pi(R^2 - r^2)(d^2 + 4R^2)} \hat{j}$$

36. For the above cylinder show that the magnetic field inside the hole is uniform. Find its magnitude and direction.

$$\text{Answer : } \vec{B} = \frac{\mu_0}{2\pi} \frac{I d}{(R^2 - r^2)} \hat{j}$$

37. A current carrying closed loop in the form of a quarter circle of radius R in y - z as well as in x - z plane with centre at origin and straight line in x - y plane as shown in figure. Find the magnitude of magnetic field at the origin.



$$\text{Answer : } B = \frac{\mu_0 i}{4\pi R} \sqrt{\left(\frac{\pi^2}{2}\right) + 4}$$

38. Two wires P and Q of same area of cross-section and lengths 0.3 m and 0.1 m respectively, are join in parallel to form a circular loop. When the two ends are connected with a battery to pass a total current 8 amp through the wires, the net magnetic field at the centre of the circular loop is $B = \pi^2 \mu\text{T}$ due to loop. Find the ratio of specific resistance of the wire P to wire Q .

$$\text{Answer : } \rho_P / \rho_Q = 5/9$$

39. If magnetic field due to long, uniform, cylindrical, current carrying conductor of radius R at a distance x below the surface (inside the conductor) is η times less in magnitude then compared to magnetic field at a distance x above the surface (outside the conductor), then find x .

$$\text{Answer : } x = R \sqrt{\frac{(\eta - 1)}{\eta}}$$

40. A solenoid of length 40 cm, radius 30 cm and total number of turns 1000 is placed near an infinite current carrying plane with its axis parallel to plane and perpendicular to current in the plane. Current per unit width of the plane is 10 amp. For what current in the solenoid, the resultant magnetic field at the centre of the solenoid will become zero. With this current in the solenoid when the solenoid is turned through 90° so that its axis becomes perpendicular to current carrying plane, what will be the resultant magnetic field at the centre of solenoid?

$$\text{Answer : } 5 \text{ mA, } 2\sqrt{2} \pi \mu\text{T}$$