ELECTROMAGNETIC INDUCTION

1. **A wire in the form of a circular loop of radius r lies in a plane normal to a magnetic field B. If this wire is pulled to take a square shape in the same plane in t seconds, find the average induced emf in the loop.**

Answer : $\pi\left(1-\frac{\pi}{4}\right)r^2\frac{B}{t}$

2. **A solenoid of self inductance 10 H and resistance 2 Ω is connected to a 10 V battery. What time will it take for the magnetic field energy stored in the solenoid to reach $\left(\frac{1}{4}\right)^{th}$ of its maximum value.**

Answer : 3.5 s.

3. **Two conducting parallel wires AL and BM at a separation l are connected by a resistance wire AB of resistance R. There exists a magnetic field B perpendicular to plane containing. Another conducting wire CD is placed on the two wires connecting them perpendicularly. Find the power needed to slide the wire CD with constant velocity v on the wires AL and BM perpendicular to CD.**

Answer : $P = \frac{B^2 l^2 v^2}{R}$

4. **A long solenoid having 1000 turns per cm carries an alternating current of peak value 1 amp. A search coil having a cross-sectional area of 10^{-4} m^2 and 20 turns is kept in the solenoid so that its plane is perpendicular to the axis of the solenoid. The search coil registers a peak voltage of 2.5×10^{-2} V. Find the angular frequency of the current in the solenoid.**

Answer : $\omega \approx 100$ **rad/s**

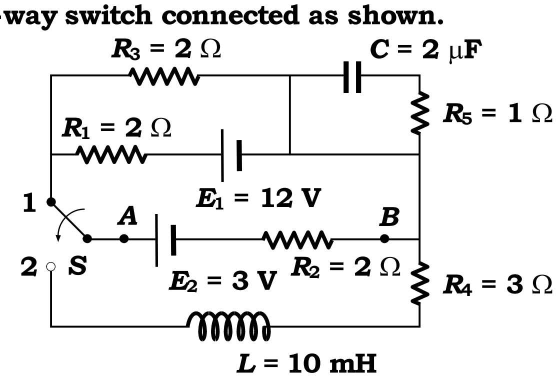
5. **Two coaxial circular loops of radii R and r with $r << R$, are separated by a distance ℓ carry currents i_1 and i_2 respectively. Estimate their mutual inductance. Also find the force with which they interact.**

Answer : $\frac{\pi\mu_o r^2 R^2}{2\left(R^2+\ell^2\right)^{3/2}}; -\frac{3\pi\mu_o i_1 i_2 r^2 R^2}{2\left(R^2+\ell^2\right)^{5/2}}\ell$

6. **A capacitor of capacitance 2 μF is charged to a potential difference of 12 V. It is then connected across an inductor of inductance 0.6 mH. Find the current in the circuit at a time when the potential difference across the capacitor is 6 V.**

Answer : $i = 6$ **amp**

7. **A circuit with two-way switch connected as shown.**

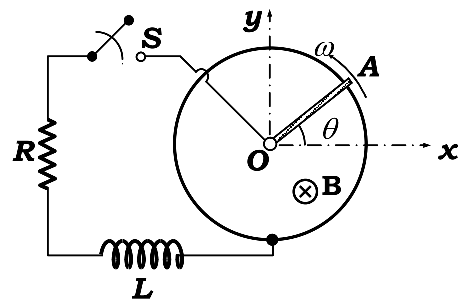


a) **When the switch is in position 1, find the potential difference $V_A - V_B$ and the rate at which joule heat is produced in R_1.**

b) If now the switch is put in position 2 at $t = 0$ find
  i) steady state current in R_4
  ii) and the time at which current in R_4 reaches half of its steady state value.

Answer : a) -5 V, 24.5 W; b) i) 0.6 A, ii) 1.4 ms

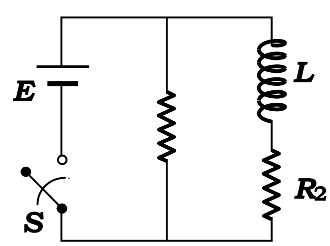
8. A conducting metal rod *OA* of mass *m* and length *l* is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end *O*. The free end *A* is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic field *B* is applied perpendicular and into the plane of rotation as shown in figure. An inductor *L* and a resistor *R* are connected through a switch *S* between the point *O* and a point C on the ring to form an electric circuit. With switch *S* open



a) Find the induced emf across the terminals of the switch.
b) The switch *S* is closed at $t = 0$
  i) Find current through resistance as a function of time.
  ii) In steady state find torque required as a function of time to maintain constant angular speed if at $t = 0$ rod is horizontal.

Answer : a) $\mathcal{E} = \frac{B\omega l^2}{2}$; b) i) $i = \frac{B\omega l^2}{2R}\left\{1 - e^{-\frac{Rt}{L}}\right\}$, ii) $\tau = \frac{B^2\omega l^4}{4R} + \frac{Mgl}{2}\cos\omega t$

9. An inductor of inductance L = 0.4 H and resistors of resistances R_1 = 2 Ω and R_2 = 2 Ω are connected to a battery of emf E = 12 V as shown in figure. The internal resistance of the battery is negligible. The switch *S* is closed at $t = 0$. What is the potential drop across *L* as a function of time? After the steady state is reached the switch *S* is opened. What is the direction and magnitude of current through R_1 as a function of time?



Answer : $V_L = 12e^{-5t}$ volts, $i_{R_1} = 3e^{-10t}$ amp.

10. A coil of inductance 1 H and resistance 10 Ω is connected to a battery of emf 50 V and negligible internal resistance at $t = 0$. Calculate the ratio of the rate at which magnetic energy is stored in the coil to the rate at which energy is supplied by the battery at $t = 0.1$ s.

Answer : $1/e$

11. Two long parallel horizontal rails, a distance d apart and each having a resistance per unit length λ, are joined at one end by a resistance *R*. A perfectly conducting rod *MN* of mass *m* is free to slide along the rails without friction. There exists a

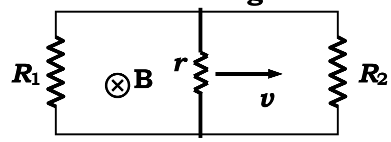
uniform magnetic field B normal to the plane of the paper, directed in to it. When a variable force F is applied, a constant current i flows through R, find

a) **the velocity of the rod and the applied force F as a function of distance x of the rod from R.**

b) **the fraction of power developed by F converted in to heat.**

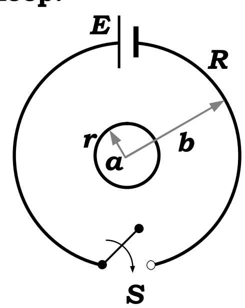
Answer : a) $v = \frac{(R+2\lambda x)}{Bd} i$, $F = iBd + \frac{2\lambda m i^2}{B^2 d^2}(R+2\lambda x)$; **b)** $\frac{B^3 d^3}{B^3 d^3 + 2\lambda m i (R+2\lambda x)}$

12. A rectangular loop with a sliding connector of length l is situated in a uniform magnetic field B, perpendicular to the plane of the loop. Resistance of connector is r. Two resistances of R_1 and R_2 are connected as shown in figure. Find the external force required to keep the connector moving with a constant velocity v.



Answer : $F = \frac{B^2 v l^2 (R_1 + R_2)}{r(R_1 + R_2) + R_1 R_2}$

13. Two concentric, coplanar, circular loops of radii a and b with $a << b$ and resistance R and r are placed as shown. At $t = 0$ the switch S is opened. Find the total charge circulating through the inner loop.

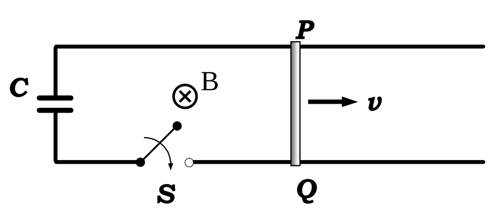


Answer : $q = \frac{\pi \mu_o a^2 E}{2bRr}$

14. Two concentric, coplanar loops of radius R, r (<<R) are made of wire with resistance per unit length λ. A time varying emf $E = \alpha + \beta t$ is applied to the larger loop. Find the induced current in the smaller loop.

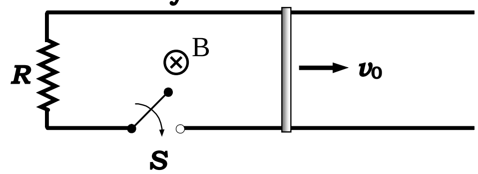
Answer : $i = \frac{\mu_o \beta r}{8\pi R^2 \lambda^2}$

15. A rod PQ of length l and resistance R is moving with a uniform speed v, in a uniform magnetic field B, directed in to the paper. A capacitor of capacitance C is connected as shown. At $t = 0$ switch S is closed. Find the charge on the capacitor as a function of time.



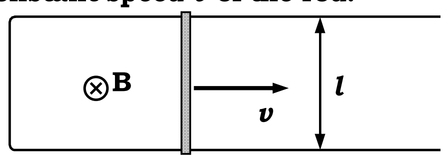
Answer : $q = BvlC\left(1 - e^{-\frac{t}{RC}}\right)$

16. A conducting rod of length l is given a velocity v_0, in a uniform magnetic field B, directed in to the paper. A resistor of resistance R is connected as shown. At $t = 0$ switch S is closed. Find the velocity of rod as the function of time



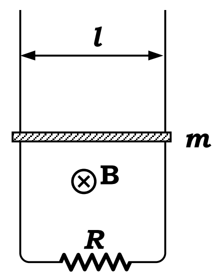
Answer : $v = v_0 e^{-\frac{B^2 l^2}{mR} t}$

17. A rod of length l and resistance R is moved left wards on a stationary perfectly conducting U shaped wire as shown in figure. What force is required to be applied on the rod to maintain constant speed v of the rod.



Answer : $F = \frac{B^2 l^2 v}{R}$

18. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length l and the resistance of the circuit is R. If a uniform magnetic field B is present in a direction perpendicular to the plane of the frame, find the terminal speed of the wire as it falls under the force of gravity.

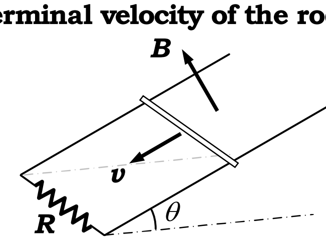


Answer : $v_T = \frac{mgR}{B^2 l^2}$

19. A non-conducting ring of mass m and radius R, having a charge q uniformly distributed over its circumference, is placed on a rough horizontal surface. A vertical, time varying magnetic field $B = kt^2$ is switched on at $t = 0$. If ring starts rotating at $t = t_0$, find the coefficient of friction between the ring and the table.

Answer : $\mu = \frac{qRkt_0}{mg}$

20. A copper rod of mass m slides under gravity on two smooth parallel rails l distance apart and set at an angle θ to the horizontal. At the bottom, the rails are joined by a resistance R. There is a uniform magnetic field B perpendicular to the plane of the rails as shown. Find the terminal velocity of the rod.

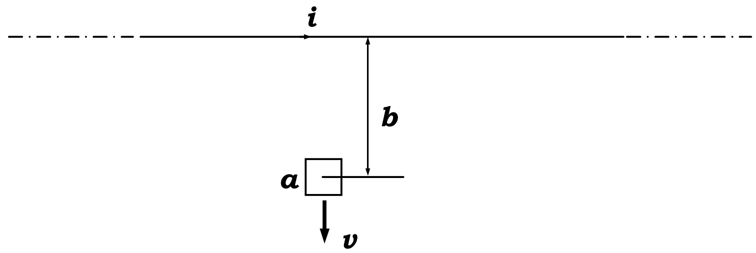


Answer : $v_T = \dfrac{mgR\sin\theta}{B^2 l^2}$

21. **A small square loop of side a and resistance r, in the plane of a long straight wire carrying current i as shown in figure is moving away with a uniform velocity v perpendicular to wire. When it is at a distance b from the wire find the induced current in the loop, if**

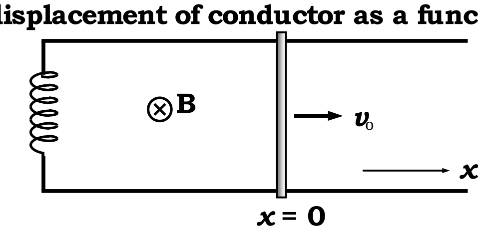
a) b is of the order of a,

b) b is very small compared to a (b >> a).



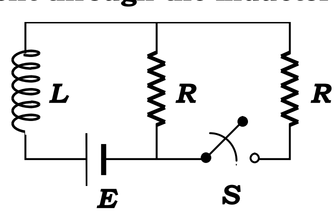
Answer : a) $i_{ind} = \dfrac{\mu_0}{2\pi}\dfrac{iva^2}{\left(b^2 - a^2/4\right)R}$ **; b)** $i_{ind} = \dfrac{\mu_0}{2\pi}\dfrac{iva^2}{b^2 R}$

22. **A conductor of mass m and negligible resistances is placed perpendicularly on smooth pair of horizontal rails separated by a distance d. An ideal inductor of inductance L is connected between the pair of rails at one end. A uniform, upward magnetic field B is present in the region. At $t = 0$ the conductor is given a horizontal velocity v_0 along the rails away form the inductor in the positive direction of x-axis as shown. Find the displacement of conductor as a function of time.**



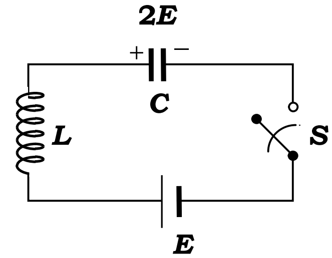
Answer : $x = \dfrac{v_0\sqrt{mL}}{Bd}\sin\left(\dfrac{Bd}{\sqrt{L}}t\right)$

23. **In the adjoining diagram after keeping the switch open for long time it is closed at $t = 0$. The emf of the cell is E and the inductor is ideal with coefficient of self induction L. Find the current through the inductor as a function of time.**



Answer : $i = \dfrac{E}{R}\left(2 - e^{-\frac{Rt}{2L}}\right)$

24. **In the adjoining diagram, the capacitor is charged to a potential $2E$ with polarities shown. The switch is closed at $t = 0$. Find the charge on the capacitor as function of time.**

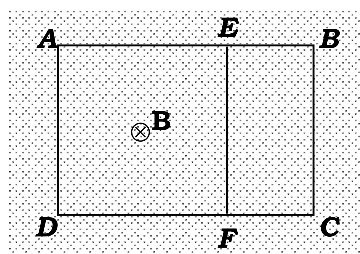


Answer : $q = CE\left(1 + \sqrt{2}\cos\left(\frac{t}{\sqrt{LC}}\right)\right)$

25. **A rectangular, conducting loop of height *l*, width *w*, mass *m* and resistance *R*, is held with its plane vertical and bottom edge just above a horizontal plane below which there exists a uniform horizontal magnetic field *B* perpendicular to the plane of the loop. The loop is dropped from rest at $t = 0$. Obtain the expression for velocity of the loop as a function of time during which it enters in to the magnetic field.**

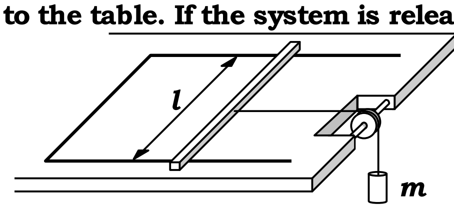
Answer : $v = \frac{mgR}{B^2w^2}\left(1 - e^{-\frac{B^2w^2}{mR}t}\right)$

26. **A rectangular frame *ABCD*, made of a uniform metal wire, has a straight connection between *E* and *F* made of the same wire, as shown in figure. *AEFD* is a square of side 1 m, and *EB* = *FC* = 0.5 m. The entire circuit is placed in a steadily, increasing, uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1 T/s. The resistance per unit length of the wire is 1 Ω/m. Find the magnitude and the directions of the currents in the segments *AE*, *BE* and *EF*.**



Answer : $\frac{5}{22}$ amp, $\frac{1}{11}$ amp and $\frac{7}{22}$ amp

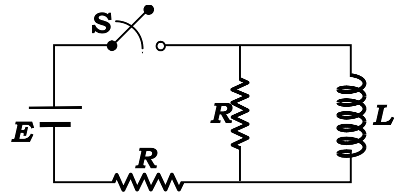
27. **A pair of smooth, parallel, horizontal, conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is *l*. A conducting massless rod of resistance *R* can slide on the rails. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass *m*, tied to the other end of the string hangs vertically. A constant magnetic field *B* exists perpendicular to the table. If the system is released from rest, find**



a) the terminal velocity achieved by the rod, and
b) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

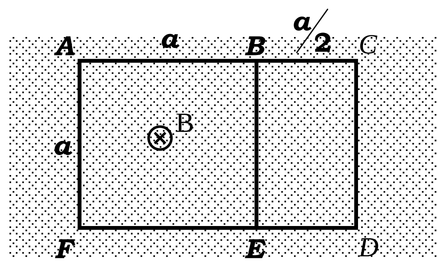
Answer : a) $v_T = \frac{mgR}{B^2l^2}$; b) $a = g/2$

28. **In the following circuit find the current flowing through the inductor of self inductance L as a function of time after the switch S is closed as $t = 0$.**



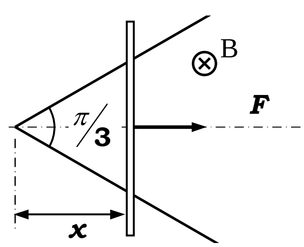
Answer : $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{2L}}\right)$

29. **Determine the current in the conductor BE of the circuit shown in the figure if the inductance of the homogeneous magnetic field is perpendicular to the plane of the drawing and changes with time according to the law $B = kt$. The resistance per unit length of the conductor is r.**



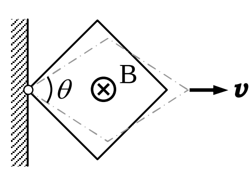
Answer : $i = \frac{ka}{22r}$

30. **A highly conducting rod moves on the smooth V shaped uniform rod of resistance per unit length ρ keeping perpendicular to the angular bisector as shown. Find the force F needed to keep the rod moving with uniform velocity v as a function x.**



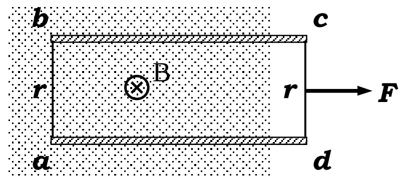
Answer : $F = \frac{B^2 v}{\sqrt{3}\rho} x$

31. **A square frame of side a and total resistance R is fixed at one corner with its plane vertical in a horizontal uniform magnetic field of strength B. The opposite corner of the square is pulled with the help of an insulating thread with uniform velocity v to deform the square. Find the magnitude and direction (clockwise or anticlockwise) of induced current in the frame as a function of angle θ in the following diagram.**



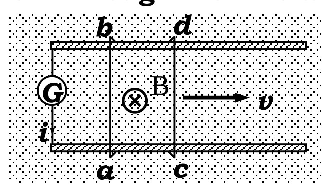
Answer : $aBv\frac{\cos\theta}{\sin\left(\theta/2\right)}$, **clockwise for** $\left(\theta < \pi/2\right)$ **anticlockwise otherwise.**

32. Two identical resistance wires *ab* and *cd* of resistance *r* and length ℓ each are joined by perfect conductors *bc* and *ad* to form a rectangle *abcd* as shown in figure. This rectangle is placed in a uniform magnetic field *B* perpendicular to its plane with side *cd* outside the field as shown. What force *F* is needed to pull the frame out of the field with uniform velocity *v*. What is the potential difference $V_c - V_d$ and $V_b - V_a$ in this situation.



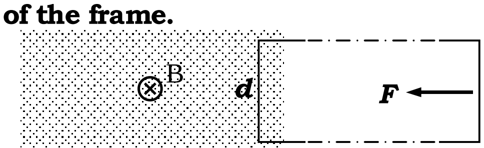
Answer : $F = \frac{B^2 v \ell^2}{2r}$, $V_c - V_d = \frac{Bv\ell}{2}$, $V_b - V_a = \frac{Bv\ell}{2}$

33. The current generator *G* circulates a constant current *i* through the circuit. Two wires *ab* and *cd* of resistance *r* and 2*r* respectively are placed on smooth, thick pair of rails at a separation ℓ with *ab* fixed and *cd* moving with constant velocity *v* towards right. Find the current through the wire *ab*.



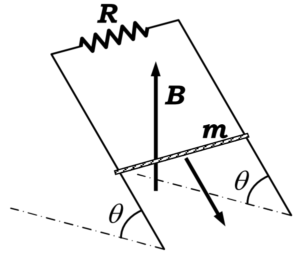
Answer : $\frac{2ir - Bv\ell}{3r}$

34. A rectangular wire frame of mass *m*, resistance *R*, of large length and width *d*, at rest with one of its smaller sides inside a uniform magnetic field *B* perpendicular to its plane. The frame is being pushed in to field by applying a constant force *F* at *t* = 0 as shown in figure. Find the velocity of frame as a function of time. Also find the terminal velocity of the frame.



Answer : $v = \frac{RF}{B^2 d^2}\left(1 - e^{-\frac{tB^2d^2}{mR}}\right)$; $v_t = \frac{RF}{B^2 d^2}$

35. A conductor of mass *m* slides down two smooth conducting parallel rails with separation *l*, inclined at angle θ from horizontal in presence of uniform vertical magnetic field *B* as shown in figure. The rails are connected by a resistance *R* at the top. Find the velocity of conductor as a function of time, hence show that if parallel rails are of sufficient length the sliding conductor will attain a steady state velocity. Hence or otherwise find that steady state velocity.

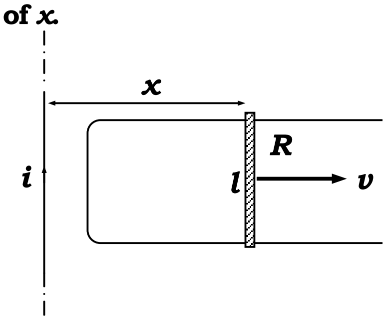


Answer : $v_T = \dfrac{mgR\sin\theta}{B^2l^2\cos^2\theta}$

36. If resistance is replaced by a capacitor of capacitance C in the above problem find the acceleration of the sliding conductor.

Answer : $\dfrac{mg\sin\theta}{m+\left(CB^2l\cos^2\theta\right)}$

37. A U shaped conductor is placed in the plane of the long straight current carrying conductor carrying a current i. Another conductor of resistance R is moved on the U shaped conductor with a uniform velocity v. Find the induced current in the loop thus form as a function of x.

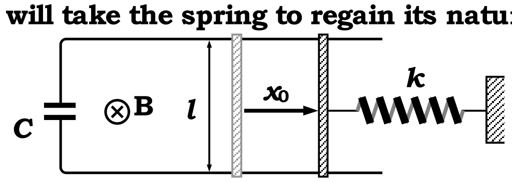


Answer : $\dfrac{\mu_o}{2\pi}\dfrac{ilv}{Rx}$

38. A magnetic flux through a stationary loop with a resistance R varies during the time interval T as $\varphi = at(T-t)$. Find the amount of heat generated in the loop during that time. The self inductance of the loop is to be neglected.

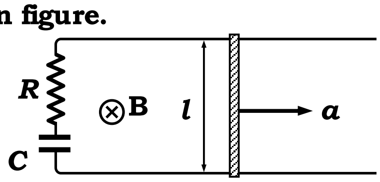
Answer : $H = \dfrac{a^2T^3}{3R}$

39. In the arrangement shown in figure a smooth conductor is placed perpendicularly on the pair of conducting rails separated by a distance l and connected to an unstretched spring of spring constant k whose other end is fixed. The rails are joined by a capacitor of capacitance C. At $t = 0$ the conductor is moved towards right compressing the spring by x_0 and released. Neglecting self inductance of the loop, find the time it will take the spring to regain its natural length.



Answer : $t = \dfrac{\pi}{2}\sqrt{\dfrac{m+B^2l^2C}{k}}$

40. A connector placed on two parallel rails located in a uniform magnetic field is pulled with constant acceleration a from rest. Find the instantaneous charge on the capacitor as shown in figure.



Answer : $q = BlaC\left\{t - RC\left(1-e^{-\frac{t}{RC}}\right)\right\}$