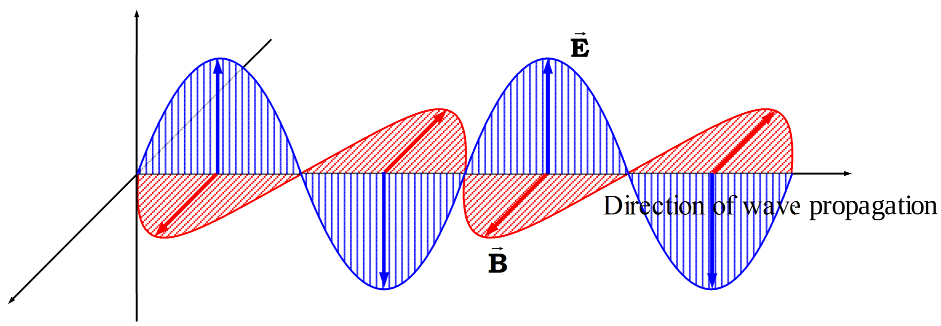
ELECTRO-MAGNETIC WAVES

Electromagnetic waves : Non mechanical waves, involving no material particles and capable of traveling through free space are called electromagnetic waves.
These are the waves in which electric and magnetic field vectors are mutually perpendicular and also perpendicular to the direction of propagation of wave. Their magnitudes vary sinusoidally keeping both electric and magnetic fields in phase.



Laws of electric and magnetic fields :

Lorentz's Force :

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Gauss's theorem for electric field :

$$\oint \vec{E}.d\vec{s} = \frac{q_{enclosed}}{\epsilon_o}$$

Gauss's theorem for magnetic field :

$$\oint \vec{B}.d\vec{s} = 0$$

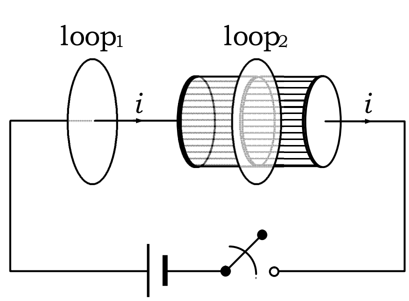
Faraday's law of electromagnetic induction :

$$\oint \vec{E}.d\vec{l} = \frac{d\varphi_B}{dt}$$

Ampere's circuital law :

$$\oint \vec{B}.d\vec{l} = \mu_o i$$

Displacement current :



According to Ampere's circuital law

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o i$$

But when we apply this on $loop_1$ the integral is non zero as the current carrying wire is piercing through it, where as for the $loop_2$ it is zero as there is no current due to charge transfer is threading through the $loop_2$. This leads to discontinuity of the current in the closed path of circuit. Maxwell argued that the time varying electric flux bounded by the surface of the $loop_2$ has the similar properties of the flow of current. As

$$\varphi_E = EA = \frac{\sigma}{\epsilon_o} A = \frac{Q}{\epsilon_o}$$

$$\Rightarrow \quad \frac{d\varphi_E}{dt} = \frac{1}{\epsilon_o}\frac{dQ}{dt} = \frac{i}{\epsilon_o}$$

$$\Rightarrow \quad \epsilon_o \frac{d\varphi_E}{dt} = i$$

$$\Rightarrow \quad \epsilon_o \frac{d\varphi_E}{dt} = \frac{dQ}{dt} = C\frac{dV}{dt}$$

Hence the quantity $\epsilon_o \frac{d\varphi_E}{dt}$, together with flow of charge in the lead wires make the current in the closed path of circuit continuous. Therefore Maxwell called $\epsilon_o \frac{d\varphi_E}{dt}$ the displacement current. Individually the conduction current $\left(\frac{dq}{dt}\right)$ or displacement current $\left(\epsilon_o \frac{d\varphi_E}{dt}\right)$ may not be continuous but their sum $\left(\frac{dq}{dt} + \epsilon_o \frac{d\varphi_E}{dt}\right)$ is always continuous along any closed path.

<u>Maxwell's equations</u> :

With idea of displacement current Maxwell modified the Ampere's circuital law as

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o \left(i_c + i_d\right)$$

$$\Rightarrow \quad \oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o \left(\frac{dq}{dt} + \epsilon_o \frac{d\varphi_E}{dt}\right)$$

This relation is called Ampere-Maxwell equation. Now the five relations governing the entire electrodynamics are as follows

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = \frac{q_{enclosed}}{\epsilon_o}$$

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = 0$$

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \frac{d\varphi_B}{dt}$$

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o \left(i + \epsilon_o \frac{d\varphi_E}{dt}\right)$$

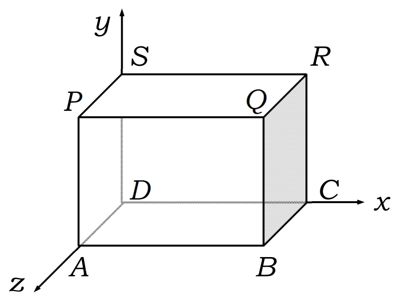
<u>Maxwell's predictions</u> :

In 1865 with the help of his theoretical work, Maxwell predicted the existence of electromagnetic waves. He predicted that

i) When a charge particle is in accelerated motion it produces electromagnetic radiations (waves).

ii) Electromagnetic waves propagate through free space with velocity 3×10^{8} m/s.
iii) The nature of electromagnetic waves is transverse.
iv) Light is an electromagnetic wave.

Electromagnetic waves are transverse :
Let us consider a plane electromagnetic wave traveling along *x*-axis. To establish transverse nature of electromagnetic waves, we must show that $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to *x*-axis (the direction of propagation of wave). Let us consider an elemental cuboid *ABCDPQRS* with sides along *x*, *y* and *z* axis respectively.



Since net charge enclosed by the cuboid is zero, therefore

$$\oint_{\text{cuboid}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} = 0$$

$$\Rightarrow \left(\int_{\text{BCRQ}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} + \int_{\text{ADSP}} \vec{\mathbf{E}}.d\vec{\mathbf{s}}\right) + \left(\int_{\text{ABCD}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} + \int_{\text{PQRS}} \vec{\mathbf{E}}.d\vec{\mathbf{s}}\right) + \left(\int_{\text{ABQP}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} + \int_{\text{DCRS}} \vec{\mathbf{E}}.d\vec{\mathbf{s}}\right) = 0 \quad \ldots (1)$$

Since it is a plane wave hence, electric as well as magnetic fields depend upon *x* and *t* but not on *y* and *z* therefore electric flux normal to *y* and *z* axes will cancel out in pair

$$\int_{\text{ABCD}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} + \int_{\text{PQRS}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} = 0 \text{, and} \quad \ldots (2)$$

$$\int_{\text{ABQP}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} + \int_{\text{DCRS}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} = 0 \quad \ldots (3)$$

from (1), (2) and (3)

$$\int_{\text{BCRQ}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} + \int_{\text{ADSP}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} = 0 \quad \ldots (4)$$

Let E_{x_1} and E_{x_2} are the magnitude of component of electric field on faces *BCRQ* and *ADSP* which are perpendicular to *x*-axis, then

$$\int_{\text{BCRQ}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} = E_{x_1}S \quad \text{and} \quad \int_{\text{ADSP}} \vec{\mathbf{E}}.d\vec{\mathbf{s}} = -E_{x_2}S$$

The negative sign is attributed to the fact that the area vector of face *BCRQ* is opposite to the area vector at the face *ADSP*. Hence from (4) as $S \neq 0$

$$E_{x_1}S - E_{x_2}S = 0$$

$$\Rightarrow \quad E_{x_1} = E_{x_2}$$

This suggests static electric field, but the static electric field can not cause the wave to propagate along the x-axis, hence

$$E_{x_1} = E_{x_2} = 0$$

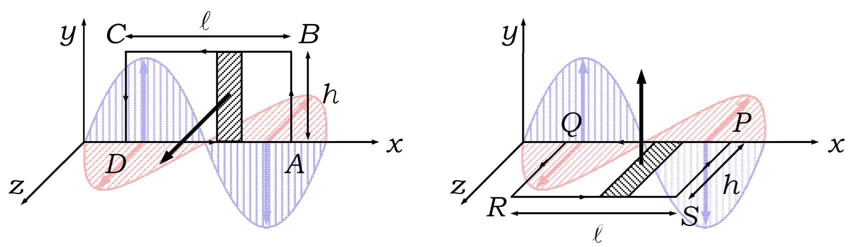
Means there is no component of electric field along the *x*-axis (the direction of propagation of the electromagnetic wave). Similar treatment for magnetic field establishes that no component of magnetic field exists along the *x*-axis. Hence the electric and magnetic fields are perpendicular to the direction of propagation of electromagnetic wave, which establishes the transverse nature of electromagnetic waves.

Velocity of electromagnetic waves :

Let us consider electromagnetic wave propagating along x-axis and the wave is represented by electric and magnetic field varying in x and t as

$$E_y = E_0 \sin\omega\left(t - {x}/{c}\right)$$

$$B_z = B_0 \sin\omega\left(t - {x}/{c}\right)$$



Let us consider a rectangular closed loop $ABCD$ in x-y plane as shown in figure. From Faraday's law of electromagnetic induction, as $\vec{\mathbf{B}} \parallel d\vec{\mathbf{l}}$

$$\oint_{ABCDA} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \frac{d\varphi_B}{dt} = \frac{d}{dt}\left\{\int_{ABCD} B(h\,dx)\right\}$$

$$\Rightarrow \quad \int_{AB} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + \int_{BC} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + \int_{CD} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + \int_{DA} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = h\frac{d}{dt}\left\{\int_{ABCD} B\,dx\right\} \qquad \dots (1)$$

At BC and DA $\vec{\mathbf{E}} \perp d\vec{\mathbf{l}}$, hence

$$\int_{BC} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_{DA} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = 0$$

therefore from (1)

$$\int_{AB} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + 0 + \int_{CD} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + 0 = h\frac{d}{dt}\left\{\int_{ABCD} B_0 \sin\omega\left(t - {x}/{c}\right)dx\right\}$$

$$\Rightarrow \quad \left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right)h = B_0 h \frac{d}{dt}\left\{\left({c}/{\omega}\right)\cos\omega\left(t - {x}/{c}\right)\right\}_x^{x+\ell}$$

$$\Rightarrow \quad E_0\left\{\sin\omega\left(t - \frac{x+\ell}{c}\right) - \sin\omega\left(t - \frac{x}{c}\right)\right\} = \frac{B_0 c}{\omega}\frac{d}{dt}\left\{\cos\omega\left(t - \frac{x+\ell}{c}\right) - \cos\omega\left(t - \frac{x}{c}\right)\right\}$$

$$\Rightarrow \quad E_0\left\{\sin\omega\left(t - \frac{x+\ell}{c}\right) - \sin\omega\left(t - \frac{x}{c}\right)\right\} = B_0 c\frac{d}{dt}\left\{\sin\omega\left(t - \frac{x+\ell}{c}\right) - \sin\omega\left(t - \frac{x}{c}\right)\right\}$$

$$\Rightarrow \quad E_0 = B_0 c \qquad \dots (2)$$

Now let us consider a rectangular closed loop $PQRS$ in x-z plane as shown in figure. From Ampere-Maxwell's law of electromagnetic induction, as $\vec{\mathbf{E}} \parallel d\vec{\mathbf{l}}$

$$\oint_{PQRSP} \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o \in_o \frac{d\varphi_E}{dt} = \mu_o \in_o \frac{d}{dt}\left\{\int_{PQRS} E(h\,dx)\right\}$$

$$\Rightarrow \quad \int_{PQ} \vec{\mathbf{B}}.d\vec{\mathbf{l}} + \int_{QR} \vec{\mathbf{B}}.d\vec{\mathbf{l}} + \int_{RS} \vec{\mathbf{B}}.d\vec{\mathbf{l}} + \int_{SP} \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o \in_o h\frac{d}{dt}\left\{\int_{PQRS} E\,dx\right\} \qquad \dots (3)$$

At PQ and RS $\vec{\mathbf{B}} \perp d\vec{\mathbf{l}}$, hence

$$\int_{PQ} \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \int_{RS} \vec{\mathbf{B}}.d\vec{\mathbf{l}} = 0$$

therefore from (1)

$$0 + \int_{QR} \vec{\mathbf{B}}.d\vec{\mathbf{l}} + 0 + \int_{SP} \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_o \in_o h \frac{d}{dt}\left\{ \int_{ABCD} E_0 \sin\omega\left(t - x/c\right) dx \right\}$$

$$\Rightarrow \quad \left(\vec{\mathbf{B}}_2 - \vec{\mathbf{B}}_1\right) h = \mu_o \in_o E_0 h \frac{d}{dt}\left\{\left(c/\omega\right)\cos\omega\left(t - x/c\right)\right\}_x^{x+\ell}$$

$$\Rightarrow \quad B_0\left\{\sin\omega\left(t - \frac{x+\ell}{c}\right) - \sin\omega\left(t - \frac{x}{c}\right)\right\} = \mu_o \in_o \frac{E_0 c}{\omega}\frac{d}{dt}\left\{\cos\omega\left(t - \frac{x+\ell}{c}\right) - \cos\omega\left(t - \frac{x}{c}\right)\right\}$$

$$\Rightarrow \quad B_0\left\{\sin\omega\left(t - \frac{x+\ell}{c}\right) - \sin\omega\left(t - \frac{x}{c}\right)\right\} = \mu_o \in_o E_0 c \frac{d}{dt}\left\{\sin\omega\left(t - \frac{x+\ell}{c}\right) - \sin\omega\left(t - \frac{x}{c}\right)\right\}$$

$$\Rightarrow \quad B_0 = \mu_o \in_o E_0 c \qquad \ldots (4)$$

From (2) and (4)

$$E_0 = B_0 c = \left(\mu_o \in_o E_0 c\right) c$$

$$\Rightarrow \quad c = \frac{1}{\sqrt{\mu_o \in_o}}$$

Intensity of electromagnetic waves :

Intensity of electromagnetic waves at a point is defined as the energy crossing unit area perpendicular to wave propagation per unit time at that point.

In static electric field the energy density in electric field E is

$$\tfrac{1}{2} \in_o E^2$$

In case of electromagnetic waves the electric and magnetic field vary sinusoidally, hence E is replaced by E_{rms}, then the average energy density due to electric field is

$$\tfrac{1}{2} \in_o {E_{rms}}^2$$

and the average energy density due to magnetic field is

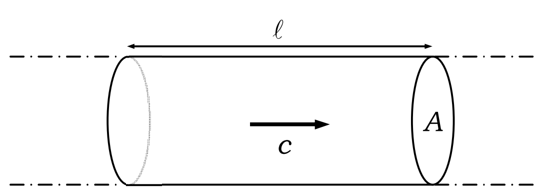
$$\tfrac{1}{2} \frac{{B_{rms}}^2}{\mu_o}$$

but $B_{rms} = \dfrac{E_{rms}}{c} = \sqrt{\mu_o \in_o} E_{rms}$, hence

$$\tfrac{1}{2} \frac{{B_{rms}}^2}{\mu_o} = \tfrac{1}{2}\left(\mu_o \in_o\right) \frac{{E_{rms}}^2}{\mu_o} = \tfrac{1}{2} \in_o {E_{rms}}^2$$

Therefore average energy density due to both the fields is

$$\tfrac{1}{2} \in_o {E_{rms}}^2 + \tfrac{1}{2} \frac{{B_{rms}}^2}{\mu_o} = \in_o {E_{rms}}^2$$



Let us consider a cylinder of area of cross-section A and length ℓ coaxial with a plane electromagnetic wave. Total energy confined in the cylinder, when electric field in electromagnetic wave is E_0

Energy density × Volume

$\Rightarrow \quad \in_o E_{rms}^2 \times A\ell$

This energy will pass the cross-section in time

$$t = \ell / c$$

Hence intensity at the cross-section is

$$I = \frac{\in_o E_{rms}^2 \times A\ell}{A \times \ell / c}$$

$$\Rightarrow \quad I = \in_o E_{rms}^2 c = \sqrt{\in_o / \mu_o}\, E_{rms}^2$$

Momentum and radiation pressure :

Max plank established that the electromagnetic radiation in desecrate in nature, and it can be considered as the energy packets called photons, whose energy depend upon the frequency of radiation (or wavelength) as follows

$$E = h\nu = \frac{hc}{\lambda} \quad \ldots (1)$$

From Einstein's theory

$$E = mc^2 \quad \ldots (2)$$

equating (1) and (2) we get

$$mc^2 = \frac{hc}{\lambda}$$

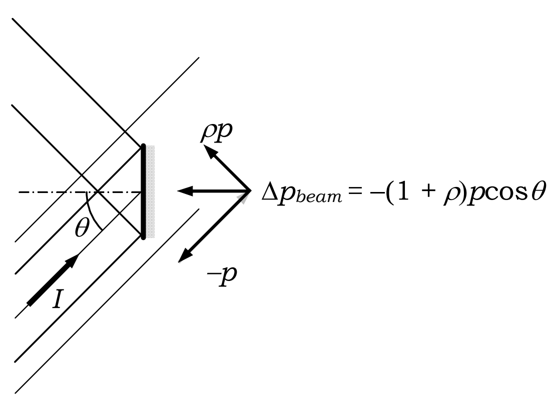
$$\Rightarrow \quad mc = \frac{h}{\lambda} = \frac{E}{c}$$

Hence momentum associated with one photon is E/c. Therefore the momentum associated with electromagnetic radiation of total energy U is given by

$$p = U/c$$

If a parallel beam of light of intensity I is incident normally on a perfectly absorbing surface of area S, the momentum transferred to the surface per unit time (Force applied by light beam on the surface) is

$$F = \frac{IS}{c}$$



If the beam is incident at an angle θ from the normal on a surface with coefficient of reflection ρ, then the incident energy and reflected energy per unit time are

$$\frac{dE_i}{dt} = IS\cos\theta, \quad \frac{dE_r}{dt} = \rho IS\cos\theta$$

normal component of change of momentum per unit time (normal component of force) is

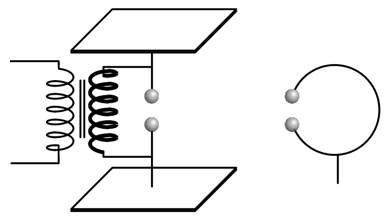
$$F_n = (1+\rho)\frac{IS\cos^2\theta}{c}$$

Therefore the radiation pressure

$$pressure = \frac{F_n}{S} = (1+\rho)\frac{I\cos^2\theta}{c}$$

Hertz Experiment :

In 1888 Hertz experimentally established the existence of electromagnetic waves. For this he used an apparatus as shown. Two large, parallel, conducting square plates separated by 60 cm are connected to two highly polished conducting spheres with thick conducting wires with a small gap between them.



The system forms an L-C circuit with small capacitance between the metal plates and the small inductance of the connecting wires. The frequency of oscillation of charge (current)in this L-C circuit is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

With this setup Hertz could obtain the electromagnetic waves of wavelength 6 m.
To detect the waves produced, a metallic ring with a small cut is used. The two end of the ring are fitted with small polished metallic spheres. When ring is placed with its plane perpendicular to the magnetic field produced by the emitter, the oscillating magnetic field produces large induced emf between the spheres connected to the ring and produce a spark between them.

Important components of electromagnetic spectrum :

N.	E-M waves	Frequency range	Produced from
1.	Gamma rays	5×10^{22} to 3×10^{18}	Nuclear reactions
2.	X-rays	3×10^{21} to 1×10^{16}	Fast decelerating electrons in X-ray tube
3.	Ultraviolet radiation	5×10^{17} to 8×10^{14}	Excited atom's electron transition
4.	Visible light	8×10^{14} to 4×10^{14}	Excited atom's electron transition
5.	Infra red radiation	4×10^{14} to 3×10^{11}	Excited atoms and molecules
6.	Micro waves	3×10^{11} to 1×10^{9}	Oscillating current in Magnetron
7.	Radio waves	1×10^{9} to 5×10^{5}	Resonant L-C circuit