MAGNETISM

1. Magnetostatics

a) Force between two magnetic poles

$$F = \frac{\mu_o}{4\pi}\frac{p_1 p_2}{r^2} \quad \text{where} \quad \mu_0 = 4\pi \times 10^{-7}\ \text{WbA}^{-1}\text{m}^{-1}$$

b) Force on a magnetic pole in a magnetic field

$$\vec{\mathbf{F}} = p\vec{\mathbf{B}}$$

where $+p$ for north pole (force in the direction of field) and $-p$ for south pole (force opposite to the direction of field).

c) Magnetic field strength due to a pole

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{p}{r^3}\vec{\mathbf{r}}$$

d) Magnetic dipole

i) Dipole moment

$$\vec{\mathbf{M}} = p\left(\vec{\mathbf{d}}\right)$$

where the $\vec{\mathbf{d}}$ is the vector starting from south pole (–ve) and terminating at north pole (+ve) in the magnetic dipole

ii) Dipole in a uniform magnetic field may or may not experience a torque but it will certainly not experience a net force.

Torque on the dipole

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{d}} \times p\vec{\mathbf{B}} = \vec{\mathbf{M}} \times \vec{\mathbf{B}} \quad \text{or} \quad \tau = MB\sin\theta$$

Work done in rotating dipole from aligned position

$$W = MB(1 - \cos\theta)$$

Potential energy of the dipole

$$U = -M.B$$

Time period of small angular oscillations of the dipole

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

iii) Dipole in a non-uniform magnetic field may or may not experience a torque but it will certainly experience a net force

Force on dipole

$$F = M\frac{dB}{dz}$$

iv) Magnetic field due to dipole

axial position

$$B = \frac{\mu_o}{4\pi}\frac{2Mr}{\left(r^2 - d^2/4\right)^2} \quad \text{If } r >> d \text{ then } B \approx \frac{\mu}{4\pi}\frac{2M}{r^3}$$

equatorial position

$$B = \frac{\mu_o}{4\pi}\frac{M}{\left(r^2 + d^2/4\right)^{3/2}} \quad \text{If } r >> d \text{ then } B \approx \frac{\mu}{4\pi}\frac{M}{r^3}$$

general position

$$B_{net} = \frac{\mu_o}{4\pi}\frac{M}{r^3}\sqrt{1 + 3\cos^2\varphi}$$

The value of angle φ for which resultant magnetic field is perpendicular to dipole moment vector.

$\varphi = \tan^{-1}\left(\sqrt{2}\right)$

2. Magnetic properties of materials

a) Bohr's magneton

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{A m}^2$$

b) Magnetic flux density (B) : It is magnetic flux per unit are.

SI unit is T (tesla) = J A^{-1} m^{-2} $[B] = [MT^{-2}A^{-1}]$

c) Magnetic Intensity or magnetizing force (H) : It is force per unit pole strength.

SI unit is N Wb^{-1} or A m^{-1}. $[H] = [L^{-1}A]$

d) Magnetic permeability (μ) : The ratio of the magnetic induction to the magnetizing field.

$$\mu = B/H$$

SI unit is N A^{-2} or Wb A^{-1} m^{-1} $[\mu] = [MLT^{-2}A^{-2}]$

e) Intensity of magnetization (I) : It is magnetic moment per unit volume.

$$I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{p_m}{V}$$

For bar magnet with pole strength m, length d and area of cross-section A

$$I = \frac{p_m}{V} = \frac{md}{Ad} = m/A$$

SI unit is A m^{-1}. $[I] = [L^{-1}A]$

f) Magnetic Susceptibility (χ) : The ratio of intensity of magnetization (I) to the magnetizing field (H).

$$\chi = I/H$$

It is a dimensionless quantity.

g) Relation between $\vec{\mathbf{B}}$, $\vec{\mathbf{H}}$ and $\vec{\mathbf{I}}$:

$$B = \mu_o (H + I)$$

h) Relation between μ_r and χ :

$$\mu_r = (1 + \chi)$$

i) Classification of Magnetic Materials

i) Diamagnetic Substances : get feebly magnetized opposite to the magnetizing field

ii) Paramagnetic Substances : get feebly magnetized in the direction of magnetizing field.

iii) Ferromagnetic Substances : get strongly magnetized in the direction of magnetizing field.

j) Curie Law of magnetism :

$$I = C\left(H/T\right) \text{ hence } \chi \propto 1/T$$

3. Earth's Magnetism

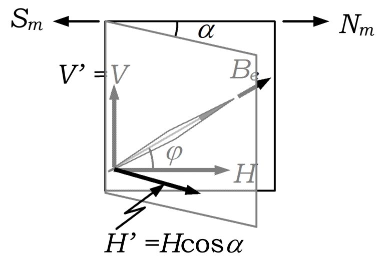
a) Magnetic meridian : The vertical plane in which magnetic north and south pole exist.

b) Angle of declination (θ) : The angle between magnetic and geographical meridian at the position of observer is called angle of declination.

c) Angle of dip (φ) : The angle between resultant magnetic field of earth and horizontal at the position of observer is called dip angle.

$$\tan\varphi = V/H \quad \text{and} \quad B_e = \sqrt{V^2 + H^2}$$

d) Apparent dip



$$\tan\varphi' = \frac{\tan\varphi}{\cos\alpha}$$

4. Lorentz Force

a) Force experienced by a charge by virtue of its charge

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

b) Since force due to magnetic field is always perpendicular to velocity of charge particle hence speed of the charge particle does not change due to magnetic field.

c) Circular motion of charge

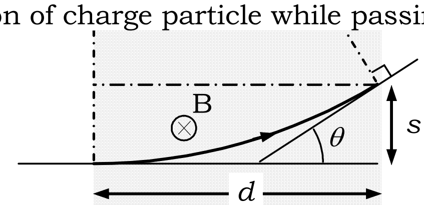
i) Radius of curvature

$$R = \frac{mv}{qB}$$

ii) time period

$$T = \frac{2\pi m}{qB}$$

iii) deflection of charge particle while passing through magnetic field



$\theta = \sin^{-1}\left(d/R\right)$ for $d < R$ and $\theta = \pi$ for $d > R$

$$s \approx \frac{d^2}{2R}$$

d) Helical motion

i) Radius of helical motion

$$R = \frac{mv\sin\alpha}{qB}$$

ii) time period

$$T = \frac{2\pi m}{qB}$$

iii) pitch of helix

$$p = \frac{2\pi mv\cos\alpha}{qB}$$

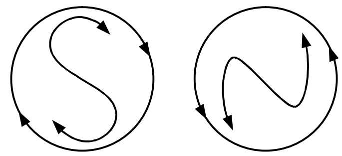
e) Net force on an arbitrary shaped current carrying wire placed in a uniform magnetic field is equal to the force on the straight wire carrying same current joining the ends of the wire.

f) Net force experienced by a current carrying closed loop in a uniform magnetic field is zero.

g) Net torque experienced by a current carrying closed loop is

$$\tau = niAB\sin\theta$$

Hence a current carrying closed loop behaves as a bar magnet placed on its axis.



Magnetic moment

i) of a current carrying loop

$$\vec{\mathbf{M}} = ni\vec{\mathbf{A}}$$

It is a vector having direction perpendicular to plane of the loop given by right hand law

ii) of charge particle rotating in a circle

$$M = \frac{q\omega R^2}{2}$$

iii) of uniformly charged spherical shell

$$M = \frac{q\omega R^2}{3}$$

iv) of uniformly charged disk

$$M = \frac{q\omega R^2}{4}$$

v) of uniformly charged solid sphere

$$M = \frac{q\omega R^2}{5}$$

vi) Ratio of magnetic moment and mechanical moment (angular momentum) of a uniformly charged object is always

$$\frac{M}{L} = \frac{q}{2m}$$

h) Force per unit length between two straight parallel current carrying conductors

$$F/l = \frac{\mu_0 i_1 i_2}{2\pi r}$$

attraction for parallel currents and repulsion for anti-parallel currents.

5. Magnetic effect of current

a) Biot-Savart's law

$$dB = \frac{\mu_o}{4\pi}\frac{idl\sin\theta}{r^2}$$

direction of magnetic field is given by right-hand Maxwell's cork-screw law.

b) Magnetic field due to various conductors

i) due to straight current carrying conductors

$$B = \frac{\mu_o}{4\pi}\frac{i}{r}(\sin\alpha + \sin\beta)$$

ii) for infinite straight conductor

$$B = \frac{\mu_o}{2\pi}\frac{i}{r}$$

iii) at the centre of current carrying circular ring

$$B = n\frac{\mu_o i}{2R}$$

iv) Magnetic field at the center of a uniform circular loop is zero if a source of emf is connected between any two points of it.

v) at a point on the axis of a current carrying ring

$$B = \frac{\mu_o niR^2}{2(R^2 + a^2)^{3/2}}$$

vi) Magnetic field at the centre due to charge rotating in a circle

$$i = qf = \frac{q\omega}{2\pi}$$

$$B = \frac{\mu_o q\omega}{2\pi R}$$

vii) Magnetic field due to long solenoid
at the centre

$$B = \mu_o \frac{N}{l} i$$

at the end of the solenoid

$$B = \mu_o \frac{N}{2l} i$$

viii) Magnetic field inside a long pipe carrying current along its length is zero.

ix) Magnetic field near an infinite current carrying plane

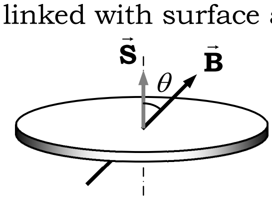
$$B = \frac{\mu_0 I}{2}$$

where I is the linear current density (current per unit width).

6. Electromagnetic Induction

a) Magnetic flux

i) Magnetic flux linked with surface area $\vec{\mathbf{S}}$ is



$$\varphi = \vec{\mathbf{B}}.\vec{\mathbf{S}} = BS\cos\theta$$

ii) unit of magnetic flux is weber.

iii) dimensional formula $[ML^2T^{-2}A^{-1}]$

b) i) Faraday and Lanze's law

$$\mathscr{E} = -\frac{d\varphi}{dt}$$

ii) direction of induced emf is such that it opposes the cause of its procuction

c) Motional emf

i) If a conductor moves in pure translation in a magnetic field then

$$\mathscr{E} = Blv$$

where B, l and v are mutually perpendicular

ii) If rod of length l rotates about an axis passing through its end and perpendicular to it

$$\mathscr{E} = \frac{B\omega l^2}{2}$$

iii) Net induced in a closed loop moving in a uniform magnetic field is zero

d) Amount of charge circulated through a closed loop in changing flux bounded by it

$$q = \frac{\Delta\varphi}{R}$$

where R is loop resistance

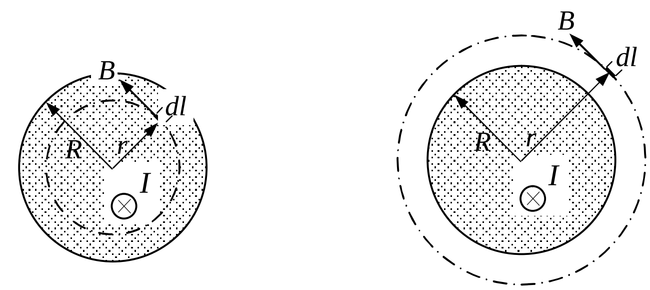
e) Amount of work done in changing flux bounded by a closed loop carrying constant current

$$w = i\Delta\varphi$$

f) Ampere's Law

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_0 i$$

Magnetic field inside and outside a long, uniform straight cylindrical wire



For $r < R$ $B = \frac{\mu_0}{2\pi}\frac{I}{R^2}r$ and for $r > R$ $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

g) Self and mutual induction

i) Coefficient of self induction $L = \varphi/i$

ii) Coefficient of mutual induction $M = \varphi_s/i_p$

iii) Coefficient of self induction of solenoid = $\frac{\mu_0 N^2 A}{l}$

iv) Unit of self/mutual induction is Henry (H)

v) Relation between self and mutual induction $M = \sqrt{L_1 L_2}$

vi) Coefficient of coupling between two coils $K = \frac{M}{\sqrt{L_1 L_2}}$

h) L-R circuit

i) Induced emf across an inductor = $L\frac{di}{dt}$

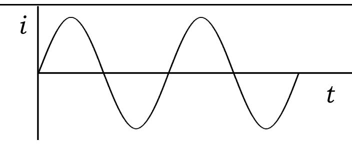
ii) Energy stored in the inductor $U = \frac{1}{2}Li^2$

iii) Current in charging L-R circuit $i = i_0\left(1 - e^{-\frac{T}{L/R}}\right)$

7. Alternating Current

a)

i) $i = i_0 \sin\omega t$ is the equation of sinusoidal alternating current.



ii) Average value of ac

$$\langle i \rangle = \frac{\int_{t_1}^{t_2} i\,dt}{t_2 - t_1}$$

from $t = 0$ to $t = T$ is zero

from $t = 0$ to $t = T/2$ is $\frac{2i_0}{\pi}$

iii) rms or effective value of current

$$i_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} i^2\,dt}{t_2 - t_1}}$$

from $t = 0$ to $t = T$ or from $t = 0$ to $t = T/2$ is

$$i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707\ i_0$$

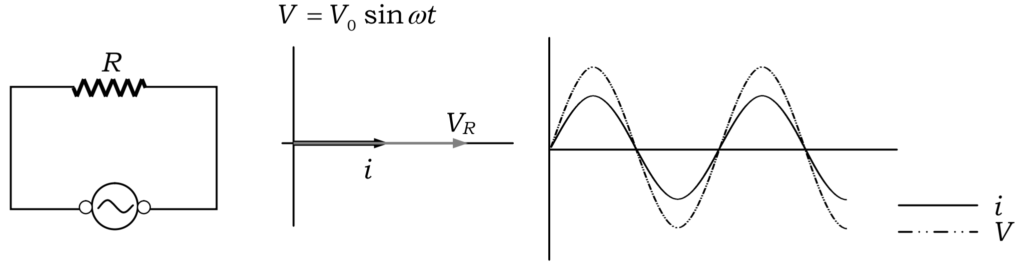
b) ac circuits

i) R-circuit

current and potential remain in same phase

$$i = i_0 \sin \omega t$$

$$V = V_0 \sin \omega t$$

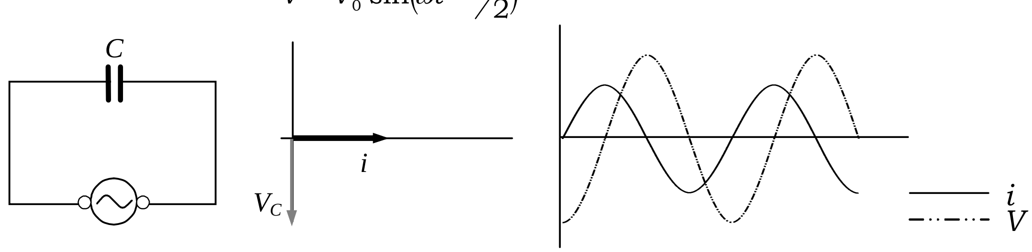


ii) C-circuit

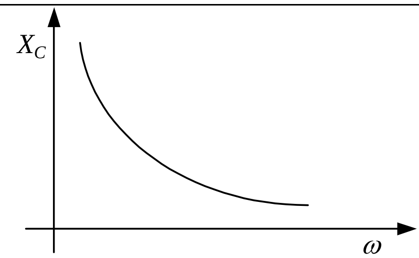
Voltage lags behind current by $\pi/2$

$$i = i_0 \sin \omega t$$

$$V = V_0 \sin\left(\omega t - \pi/2\right)$$



capacitive reactance $X_c = \frac{1}{\omega C}$ decreases with increase in ω

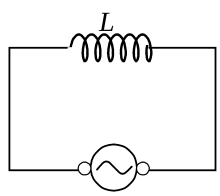


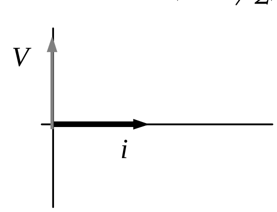
ii) L-circuit

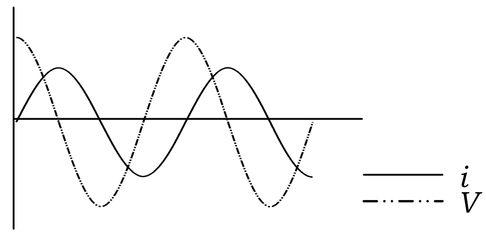
Voltage leads the current by $\pi/2$

$i = i_0 \sin \omega t$

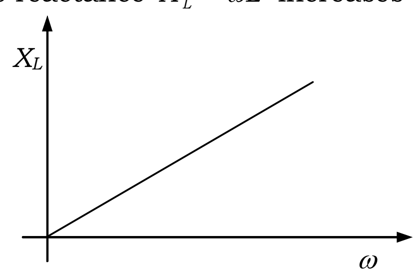
$V = V_0 \sin(\omega t + \pi/2)$







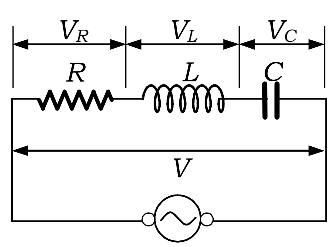
inductive reactance $X_L = \omega L$ increases with increase in ω

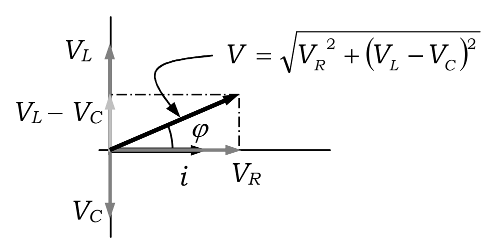


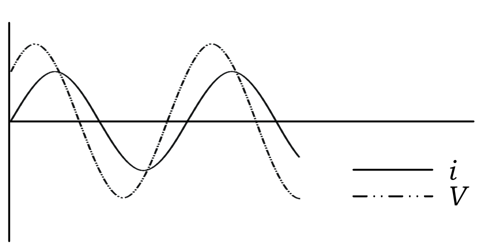
iii) RLC circuit

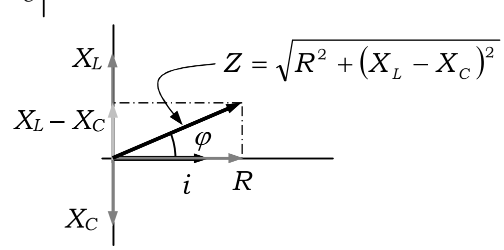
$i = i_0 \sin \omega t$

$V = V_0 \sin(\omega t + \varphi)$









$V_R = i_{rms} R$, $V_L = i_{rms} X_L$ and $V_C = i_{rms} X_C$

$\tan\varphi = \dfrac{X_L - X_C}{R}$ or $\cos\varphi = \dfrac{R}{Z}$ and

$$\tan\varphi = \frac{V_L - V_C}{V_R} \text{ or } \cos\varphi = \frac{V_R}{V}$$

iv) Resonance in series L-C circuit

at resonance $X_L = X_C$ hence

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ and } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Q-factor $Q = \frac{V_L}{V} = \frac{\omega L}{R}$

v) Resonance in parallel LCR circuit

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ and } Z = \frac{L}{RC}$$

condition for the resonance to occur

$$R < \sqrt{L/C}$$

vi) Average power in ac circuit for complete cycle

$$\langle P \rangle = \frac{i_0 V_0}{2}\cos\varphi = i_{rms} V_{rms} \cos\varphi$$

$\cos\varphi$ is called power factor of ac circuit.

In pure capacitive or inductive circuits $\varphi = \pm \pi/2$ hence $P = 0$ in these circuits and the current flowing in these circuits is called watt less current.

To reduce current in ac circuit an inductor coil in series of the circuit is connected which reduces current without appreciable loss of power and called choke coil.